Bargaining Power in Networks

Chapter 12 in Networks, Crowds, and Markets: Reasoning About a Highly Connected World.
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Summary slides by J. Musacchio

Intro

• What is “Power” in a network?

Power

• Suppose this group produces value with pairwise interactions.
• Ideas for thinking about power

1) Dependence. Recalling that social relations confer value, nodes A and C are completely dependent on B as a source of such value. B on the other hand, has multiple sources.

2) Exclusion. Related to (1), B has the ability to exclude A and C. In particular, suppose each person were to choose a “best friend” in the group, then B has the unilateral power to choose one of A and C, excluding the other. (However, B does not have the analogous power over D.)

Power Notions, cont’d

(iii) Satiation. Having diminishing rewards for increased amounts of something “rich” need an unequal share of spoils to make it worthwhile.

(iv) Betweenness. Lies on paths (and particularly short paths) between many pairs of other nodes.

Network exchange

• Typical setup
  – People are nodes on a graph
  – Potential money on edges if nodes on each side choose to start relationship
  – Negotiate on sharing of money
  – Often nodes only allowed one partner
  • "1-exchange rule"
  – Play on same graph multiple times
  – Varying information structure
    • See everything to see only local info of what’s happening

Common examples
Experimental results

• 2 node path
  – $\frac{1}{3} - \frac{2}{3}$ split common

• 3 node path
  – Middle gets 5/6
  – Ends 1/12

• 4 node path
  – B gets 7/12 to 2/3

• 5 node path
  – Middle, C only gets slightly better than ends

Experimental Results

• B favors relations with either A or C, since she has power over them

• B usually trades with a or C
  – and takes most of the money!

• D trades with E on similar footing

Experimental Results cont’d

• Like 4 node path

• B has a bit more power

• Gets a bit more money

unstable

Observation

• Network exchange setups with bi-partite graphs are like buyer seller networks

• Bi-partite
  – Nodes can be split into two “types” and all edges connect nodes of opposite type

Nash-Bargaining view

• No deal: A gets x, B gets y

• Deal: A and B split 1

• Surplus from deal s = 1 - x - y
Nash Bargaining

• Share the surplus equally
  – A gets \( x + s \) \( \frac{1}{2} \) \( (x+1-y) \)
  – B gets \( y + s \) \( \frac{1}{2} \) \( (y+1-x) \)

• Captures importance of outside options.

Aside: General Nash bargaining Concept

• More generally – Nash bargaining solution maximizes product of 2 players utility gains
  – Example: share a link with bandwidth 1
    – User 1 utility: \( v(x) = 0.2 \log x + 0.3 \log(1-x) \)
    – Max: \( 0.2(\log 1^{-1/2}) + 0.3(\log 1^{-1/2}) \)
    – \( x = 1/2, (1-x) = 1/2 \)

• Nash bargaining solution is the unique sharing that satisfies 4 axioms
  – Symmetry,
  – Pareto efficiency
  – Invariance to equivalent payoff representations (like change of units or shifts)
  – Insensitivity to irrelevant alternatives

Ultimatum

• Split a dollar
• A proposes a sharing
• B accepts or rejects
• If B rejects BOTH get nothing

Ultimatum

Theory

• A should propose 0.99 for himself, .01 for B
• B’s dominant strategy in subgame that follows is accept

Practice

• In experiments, this doesn’t happen
  – More balanced offers
  – B has “emotional cost” of being cheated

Network Exchange – Stable Outcomes

Instability: Given an outcome consisting of a matching and values for the nodes, an instability in the outcome is an edge not in the matching, joining two nodes \( X \) and \( Y \), such that the sum of \( X \)'s value and \( Y \)'s value is less than 1.

Stability: An outcome of network exchange is stable if and only if it contains no instabilities.
Balanced Outcome

Balanced Outcome: An outcome (consisting of a matching and node values) is balanced if, for each edge in the matching, the split of the money represents the Nash bargaining outcome for the two nodes involved, given the best outside options for each node provided by the values in the rest of the network.

Model

Rubinstein Stahl Bargaining

- Model Developed by Ariel Rubinstein
  - Rubinstein’s model was an infinite-horizon version of Ståhl’s finite horizon game:

Nash Equilibria

- Many
- Example
  - Player 1 demands x=1, and refuses x < 1
  - Player 2 offers x = 1, and accepts any offer
- But this is not subgame perfect
  - If player 2 offers x > δ, player 1 is compelled to accept.
Proof Strategy

- Derive upper and lower bounds on continuation payoffs on each player
- Show that upper and lower bounds are the same.
- Exploit structure of game

Proof Payoffs Notation

<table>
<thead>
<tr>
<th>Period Type</th>
<th>Player 1 Proposes rule</th>
<th>Player 2 Proposes rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1's Payoff range</td>
<td>$[v_1, \bar{v}_1]$</td>
<td>$[w_1, \bar{w}_1]$</td>
</tr>
<tr>
<td>Player 2's Payoff range</td>
<td>$[w_2, \bar{w}_2]$</td>
<td>$[v_2, \bar{v}_2]$</td>
</tr>
</tbody>
</table>

Continuation payoffs normalized so that the pie has size 1 at the beginning of the subgame.

Proof

- Suppose Player 1 makes an offer $(x,1-x)$
  - Player 2 accepts if $1 - x > \delta_2 \bar{v}_2$
  - Thus player 1 can make at least $v_1 \geq 1 - \delta_2 \bar{v}_2$
  - By symmetry $v_2 \geq 1 - \delta_1 \bar{v}_1$

Proof (cont’d)

- Suppose player 2 makes an offer $q_2$
  - Player 2 never offers more than $\delta_1 \bar{v}_1$
  - (that’s the most player 1 could do by refusing)
  - Thus $\bar{w}_1 \leq \delta_1 \bar{v}_1$
  - By symmetry $\bar{w}_2 \leq \delta_2 \bar{v}_2$

Proof (cont’d)

- Suppose Player 1 makes an offer $q_1$
  - Player 2 always rejects whenever $1 - x < \delta_2 \bar{v}_2$
  - Player 1’s maximum continuation payoff is the greater of the following two:
    - The most $P_1$ get $P_2$ to accept to in the current slot
    - The discounted maximum continuation payoff in the next slot if $P_2$ rejects.
  - $\bar{v}_1 \leq \max(1 - \delta_2 \bar{v}_2, \delta_1 \bar{v}_1) \leq \max(1 - \delta_2 \bar{v}_2, \delta_2 \bar{v}_1)$

Proof (cont’d)

- Claim: $\bar{v}_1 \leq \max(1 - \delta_2 \bar{v}_2, \delta_1 \bar{v}_1) = 1 - \delta_2 \bar{v}_2$
- Suppose not, then $\bar{v}_1 \leq \delta_2 \bar{v}_1$
  - $\bar{v}_1 = 0 \Rightarrow 1 - \delta_2 \bar{v}_2 > \delta_2 \bar{v}_1 \Rightarrow$ contradiction.
Proof (cont’d)

- Thus
  \[\bar{v}_1 \leq 1 - \delta_2 \bar{v}_2 \quad \text{and} \quad \bar{v}_1 \geq 1 - \delta_2 \bar{v}_2\]
  \[\bar{v}_2 \leq 1 - \delta_1 \bar{v}_1 \quad \text{and} \quad \bar{v}_2 \geq 1 - \delta_1 \bar{v}_1\]

- Thus
  \[\bar{v}_1 \geq 1 - \delta_2 \bar{v}_2 \geq 1 - \delta_2 (1 - \delta_1 \bar{v}_1)\]
  \[\bar{v}_1 \geq 1 - \frac{\delta_2}{1 - \delta_1 \bar{v}_1}\]
  \[\bar{v}_1 \leq 1 - \delta_2 \bar{v}_2 \leq 1 - \delta_2 (1 - \delta_1 \bar{v}_1)\]
  \[\bar{v}_1 \leq 1 - \frac{\delta_2}{1 - \delta_1 \bar{v}_1}\]
  \[\bar{v}_1 = \bar{v}_1 = 1 - \frac{\delta_2}{1 - \delta_1 \bar{v}_1}\]

Comparative Statics

Relative Patience determines shares...
- Fix \(\delta_2\), for \(\delta_1 \to 1\), P1 gets whole pie
- \(\delta_2 = 0, \delta_1 > 0\) P1 gets whole pie

- Fix \(\delta_1\), for \(\delta_2 \to 1\), P2 gets whole pie
- \(\delta_1 = 0\), P2 gets \(\delta_2\)

- If \(\delta_1 = \delta_2 = \delta\)
  - P1 gets \(\frac{1}{1+\delta}\) P2 gets \(\frac{\delta}{1+\delta}\)
  - First mover advantage disappears as \(\delta \to 1\)

SPE

- In subgames beginning with P1, P1 offers
  \[\bar{v}_1 = \bar{v}_1 = 1 - \frac{\delta_2}{1 - \delta_1 \bar{v}_1}\]

- P2 is indifferent between accepting and rejecting
  - In SPE P2 must accept
  - If he rejects \(x = \bar{v}_1\) with some probability, P1’s best response would have been to charge \(x = \bar{v}_1\)