1. You are asked to evaluate the chance of success of project in order to decide if it is a good investment. At the starting time, which we call quarter 0, the project is in the (C)onceptualization stage. In the (C)onceptualization stage your engineers are searching for an idea that is worthy of taking to the next stage. Every quarter in stage C, there is a 30% percent chance of jumping in the subsequent quarter to the next stage, called (D)evelopment, and a 70% chance of staying in the Conceptualization stage. In the (D)evelopment stage, there is a 30% chance that in the next quarter the project jumps to the (P)roductization stage, a 70% chance it stays in the (D)evelopment stage. In the (P)roductization stage, there is a 30% chance that in the next quarter the project jumps back to the (D)evelopment stage because an unforeseen problem comes up with the product idea when your engineers try to productize it. In the (P)roductization stage, there is also a 40% chance that in the next quarter the project remains in the (P)roductization stage, and a 30% chance the project moves to the success stage. If and when the project reaches the success stage, it stays there with probability 1.

(a) Describe this situation as a Markov chain. What is the transition matrix? For consistency, order the states \{C, D, P, S\}.

(b) Which states are transient, and which are recurrent?

(c) Suppose you have at most 10 quarters to reach success, before the project exhausts its budget. What is the chance of reaching success? You can give a decimal answer and use a tool like Matlab to raise your transition matrix \(p\) to an appropriate power if you chose to compute it that way.

(d) Suppose you spend $1 million every quarter you are not in the success stage, and that if and when you reach success you receive a one-time payment of $10 million. What is the expected net cash flow of the project? (We will not bother to discount future payments with a discount factor.)

2. In time slot \(i\), a transmitter chooses one channel randomly to transmit on from a set of channels numbered \{1, 2, 3\} with equal probability. In time slot \(i + 1\), the transmitter chooses with equal probability from one of the 2 channels not used in time slot \(i\). In time slot \(i + 2\) the transmitter uses the channel not used in either slot \(i\) or \(i + 1\). In time slot \(i + 3\), the transmitter returns to picking from all three channels with equal probability, and proceeds according to the same rules it followed after slot \(i\). Model the system as a Markov chain.

3. A packet transmitter alternates between 3 transmit rates – slow, medium, and fast. In each packet transmit attempt, the transmission fails with probability 0.001 if sent from the slow state, 0.01 from the medium state, and 0.1 from the fast state. Every time a packet failure happens, the transmitter slows by one level for the next packet, unless it is already in the slow state. Every time a packet is sent without failure, the transmitter speeds up one level for the next packet, unless it is already in the fast state. Describe the system as a Markov chain. Let the time index correspond to each packet send attempt. a) If at time 0 the system starts in slow state, what is the chance the system is in the fast state at time 5? What about time 10? b) If the system starts at time 0 in the fast state, what is the chance the system is in the fast state at time 5?, and time 10?