1. Consider the data clustering problem described on pages 49 and 50 of *Optimization and Algorithms*.

(a) Formulate the problem as an LP. Modify your LP to be in standard form (e.g., \( \min f'x \) s.t. \( Ax = b, x \geq 0 \)).

(b) Solve the LP in Matlab, and report your model coefficients and error.

(c) Modify the problem by adding a new data record. Client 9 spends 100, 150, 230, and 10 on the 4 respective categories. His employer pays 175. What are your new model coefficients and error?

2. Consider the following problem. An online ad-network can place ads of types I, II, and III. There are two types of users (A, B), which the network can determine by their browsing behavior. Type A users can be shown ads of type II, and III. Type B users can be shown ads of types I and II. There will be 10 and 15 million visits of user types A, B over the next week. The ad-network is allowed to show ads at most 9, 5, and 7 million times for ad types I, II, III respectively. The ad network gets paid 5 cents per showing (impression) of ad type I, and 10 and 7 cents for ad types II and III respectively. At most, 1 ad is allowed to be shown per visit. The data is summarized in the table below.

<table>
<thead>
<tr>
<th>Ad Type</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>number</th>
</tr>
</thead>
<tbody>
<tr>
<td>User Type B</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>10 million</td>
</tr>
<tr>
<td>max ads</td>
<td>9 million</td>
<td>5 million</td>
<td>7 million</td>
<td>15 million</td>
</tr>
<tr>
<td>value</td>
<td>5 cents</td>
<td>10 cents</td>
<td>7 cents</td>
<td></td>
</tr>
</tbody>
</table>

- Formulate the revenue maximization problem as an LP. Please use the following convention:
  - \( x_1 = \) ads of type II shown to users of type A
  - \( x_2 = \) ads of type III shown to users of type A
  - \( x_3 = \) ads of type I shown to users of type B
  - \( x_4 = \) ads of type II shown to users of type B

- Modify the LP to be in standard form.

- Place the problem data into a Tableau.

- Choose the following variables to be your first basis: \( \{ x_1, x_2, x_3, s_B, s_{III} \} \) where \( s_B \) is your slack variable on the constraint on the number of ads shown to users of type B and \( s_{III} \) is the slack variable on the constrained number of type III ads that can be shown. Do row operations on your tableau until the rows corresponding to this basis make an identity matrix. Also, do row operations to make the cost row have zeros for the elements of the basis.

- What is the basic solution from the above? Is it basic feasible?

- Check the cost row to see if there is a direction that the solution can be improved. If so, figure out what the adjacent basis is in that direction, and do row operations to make the tableau have an identity in the corresponding rows of the new basis, and have zeros in the cost row for elements of the new basis. Repeat if needed until there are no more directions for which cost can be reduced.

- What solution and value of the objective did you find? Is it a unique optimum? Hint: see if there are directions in which the “reduced costs” are zero. You can move in those directions while staying feasible and not change the objective.