Final Examination

Instructions: Closed book, no communications devices, 180 minutes, points as marked. Start each problem on a separate piece of paper.

1. Alice moves first, then Beth; each can either stop the game or keep it going. If Alice chooses S (stop), then the game is over and she gets payoff 1 and Beth gets 0. If Alice chooses K (keep going), then the game continues and it is Beth’s move. If Beth chooses s (stop), then she gets payoff 3 and Alice gets 0, but if Beth chooses k (keep going) then each player ends up with payoff 2.
   a. Draw the extensive form for this two player game.
      3 pts for structure and 1 pt for payoffs.
   b. Write out the strategy sets for both players, and write out the strategic form (bimatrix).
      2 pts for strategy sets and 2 pts for bimatrix
   c. Find all Nash equilibria (NE) of the game, pure and mixed.
      2 pts for pure NE (1 for BR, 1 for result) and 4 pts for mixed NE (2 for equations and 2 for result)
   d. Which (if any) of the NE are subgame perfect?
      3pts
   e. Which strategy profiles are efficient (i.e., maximize the payoff sum)?
      3pts

2. Suppose that two player game just described is repeated indefinitely, and each player’s payoffs in that repeated game is the discounted sum of the stage game payoffs; the discount factor d is somewhere between 0 and 1.
   a. What aspects of the repeated game tend to make the discount factor d closer to 1?
   b. Describe a strategy profile for the repeated game that might promote efficiency.
      4 pts.
   c. Suppose that d=0.6. Show whether your strategy profile can indeed support efficient play as a NE of the repeated game.
      6 pts. 4 for reasoning and 2 for conclusion

3. Consider the payoff matrix

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3, 3</td>
<td>0, 2</td>
</tr>
<tr>
<td>B</td>
<td>2, 0</td>
<td>2, 2</td>
</tr>
</tbody>
</table>

Please turn over
a. Find all NE of the game, including mixed (if any).
2 pts for each NE.

b. Which (if any) of these NE are payoff dominant? Risk dominant?

c. According to evolutionary game theory, which NE (if any) are stable?
3 pts. 1 for each statement

d. Which NE is most likely to approximate the outcome of an evolutionary game if played for a long time? Explain your reasoning very briefly. You may either answer (A, A) for its payoff dominance, or (B, B) for its risk dominance, but the mixed NE is not likely since it is not evolutionary stable.
3 pts, 1 for any result (AA or BB) and 2 for corresponding reasoning

4. Imagine two firms have to choose prices for related products. Firm 1 chooses its price $x$, and firm 2 chooses its price $y$. The sales firm 1 sees go down with the price it charges, but increase with the price firm 2 charges. In particular the quantities sold by firms 1 and 2 are respectively $Q_1 = 120 - x + \frac{1}{2}y$ and $Q_2 = 120 - y + \frac{1}{2}x$.

The cost per unit for firm 1 and 2 are 10 and 40 respectively, so their payoff functions are

$V_1 = Q_1(x - 10)$ and $V_2 = Q_2(y - 40)$.

Compute the values of $x$ and $y$ in Nash equilibrium.
4 pts for maximization problem, 4 pts for FOCs, 6 pts for solving the system and getting the answer

5. Consider a routing game in which a population (size normalized to 1) chooses whether to take route 0 or route 1 to a destination. Route 0 has a delay of $x$ hours, where $x$ is the fraction of the population that takes route 0. Route 1 has a fixed delay of 1 hour. Each player in the population represents a negligibly small fraction of the total population, and each player wants to minimize his delay.

a. Find the Wardrop (or Nash) equilibrium of the game.
6 pts. 3 for result and 3 for explanation.

b. Is the Wardrop equilibrium also an ESS? Show that the condition for ESS is satisfied (or not).
2 pts. 1 for “Yes” and 1 for explanation.

c. The delay for route 0 is $x$ and the delay of route 1 is 1. Since $x$ and $(1-x)$ are the fractions that take each respective route, the average delay is $x^2 + 1 - x$. 

Please turn over
What is the value of $x$ that minimizes the average delay and hence is socially optimal?
4 pts. 2 for FOC and 2 for result.

d. What is the ratio between the social optimal average delay and the Wardrop equilibrium average delay?
2 pts

e. Imagine you could charge a fixed toll for route 0. Also suppose that whole population values their time at $50 per hour. (e.g. it’s worth spending $50 to reduce delay by 1 hour.) What toll would you charge on route 0 to induce the socially optimal traffic pattern?
4 pts.

6. Consider a game between an (I)ncestent firm and a potential new entrant (S)tart-up firm in the market for widgets. Nature endows the incumbent firm with one of 2 possible “types” of costs for producing widgets. Specifically, the incumbent has (L)ow costs with probability $p$ or (H)igh costs with probability $1-p$. The incumbent knows the actual type, while the potential start-up only knows the probabilities. The incumbent chooses whether to price their widgets (M)oderately or (P)ricey. Observing this pricing, the start-up decides whether to (E)nter or (N)ot enter the market. Entering the market when the incumbent has low costs always results in the start-up losing money, while entering the market when the incumbent has high costs always results in the start-up making money. The incumbent is always worse off when the start-up enters. The payoffs of specific outcomes are as follows.

<table>
<thead>
<tr>
<th>Incumbent Type</th>
<th>Incumbent Price</th>
<th>Start-up Entry Decision</th>
<th>Payoff $(I, S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>M</td>
<td>E</td>
<td>$(1, -1)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N</td>
<td>$(1.5, 0)$</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>E</td>
<td>$(1, -1)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N</td>
<td>$(2, 0)$</td>
</tr>
<tr>
<td>H</td>
<td>M</td>
<td>E</td>
<td>$(-1.5, 1)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>E</td>
<td>$(-1, 1)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N</td>
<td>$(1.5, 0)$</td>
</tr>
</tbody>
</table>

a) Draw the extensive form game tree. Make sure to draw any information sets correctly.

4pts Structure, info set, payoff and nature
b) Write out the incumbent’s strategy set. Which strategies are separating and which are pooling (if any)?
   2pts for strategy set. 1 for separating and 1 for pooling.

c) Write out the entrant’s strategy set. How many pure strategies does she have?
   2pts for strategy set and 2pts for counting

d) Is there a Perfect Bayesian Nash Equilibrium (PBE) in which the incumbent follows a separating strategy? If so, describe the strategies of each player, and the beliefs of the start-up after observing (M)oderate or (P)ricey being played by his opponent. If not, explain why.
   2pts for the conclusion and 2pts for explanation.

e) Is it possible to have a Perfect Bayesian Nash Equilibrium (PBE) in which the incumbent follows the pooling strategy of always playing (P)ricey? If so, does this PBE exist for all values of $p$? If not, find the range of $p$ values for which this PBE exists. (4pts)