1. Taking your chances in love:

(a) Draw the EFG.

(b) Find all sub-games of this EFG.

There are three subgames, the ones that start with B’s nodes.

(c) Find a BNE that is SGP when \( p = \frac{1}{3} \) and \( q = \frac{1}{3} \).

B1 will always accept, B2 and B3 will always reject. Knowing this and the fact that the three games are now equally likely, the expected payoff to A of Asking,

\[
E_A(\text{Ask}) = \frac{1}{3} \times 5 + \frac{1}{3} \times (-1) + \frac{1}{3} \times (-10) = -2
\]

\[
E_A(\text{Not}) = 0
\]

It is better for A to not ask than to ask. So the BNE will be when A does not ask.

(d) Find the conditions on \( p \) and \( q \) that are necessary for it to be BNE for A to ask B out.

\[
E_A(\text{Ask}) = p \times 5 + q \times (-1) + (1 - p - q) \times (-10)
\]

\[
E_A(\text{Not}) = 0
\]

For Ask to be the BNE, we need: \( E_A(\text{Ask}) > E_A(\text{Not}) \).

\[
\Rightarrow 5p - q + (1 - p - q) \times (-10) > 0
\]

\[
\Rightarrow 15p + 9q > 10
\]
2. Market for lemons:

Given value of the car is uniformly distributed over $0 - $5000 and $V_B = 1.5V$
Thus the expected value of the car for the buyer $E[V] = \frac{1}{2} \times 5000$, and the bid they are willing to make $= 1.5 \times \frac{1}{2} \times 5000 = \frac{3}{4} \times 5000$.

Knowing this all the cars with value more than $\left(\frac{3}{4}\right) \times 5000$ will leave the market. Now the value of the car is uniformly distributed over $0 - \frac{3}{4} \times 5000$.
Knowing this the expected value of the car for the buyer $E[V] = \frac{2}{8} \times 5000$, and the bid they are willing to make $= \left(\frac{3}{4}\right)^2 \times 5000$.

Knowing this all the cars with value more than $\left(\frac{3}{4}\right)^2 \times 5000$ will leave the market, and the value of the car is uniformly distributed over $0 - \frac{3}{4} \times \left(\frac{3}{4}\right)^2 \times 5000$.

And in $n$ rounds thinking and belief updates the highest value of the cars in the market will be $\left(\frac{3}{4}\right)^n \times 5000$.

As $n \to \infty$ value of the car in the market $= 0$, or in other words there will be no market.

[continued...]
3. The dangerous game of signaling:

(a) Extensive Form Game

(b) Separating strategy:
Player A is can play a separating strategy if B has the following posterior belief:
- a – probability of Fit A after seeing signal played and is uninjured, and
- b – probability of Fit A after seeing not played;
are such that (a=1 and b=0) or (a=0 and b=1). Let’s test if any of these lead to a BNE.

Case I:
B’s posterior belief: a = 1, b=0 (All who Play and are Uninjured are Fit. All who Not play are Unfit.)

B’s expected payoffs based on belief at the information sets:

At B1, strategy: reject at B1

At B2, after seeing Play (uninjured): infer fit A
expected payoff of accept= 10 and
expected payoff of reject= 0, thus accept at B2.

At B3, after seeing Not played: infer unfit A
expected payoff of accept= -10 and
expected payoff of reject= 0, thus reject at B3.

B’s strategy consistent with belief:
Accept A if they play and are uninjured. Reject A if they don’t play.

A’s strategy based on the table 1 of expected payoffs below, the best response of A is:
Play if Fit and Not play if Unfit.

This is consistent with the beliefs of B, so this game will lead to a PBNE.
Table 1: Expected Payoffs of A

<table>
<thead>
<tr>
<th></th>
<th>Play</th>
<th>Not</th>
<th>Scratch work: Payoff of Play</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit A</td>
<td>7</td>
<td>0</td>
<td>$0.1 \times (-20) + 0.9 \times 10$</td>
</tr>
<tr>
<td>Unfit A</td>
<td>-5</td>
<td>0</td>
<td>$0.5 \times (-20) + 0.5 \times 10$</td>
</tr>
</tbody>
</table>

Case II:
B’s posterior belief: $a = 0$, $b=1$ (All who Play and are uninjured are Unfit. All who Not play are Fit.)

B’s expected payoffs based on belief at the information sets:
At B1, strategy: reject at B1

At B2, after seeing Play (uninjured): infer Unfit A expected payoff of accept= -10 and expected payoff of reject= 0, thus reject at B2.

At B3, after seeing Not played: infer Fit A expected payoff of accept= 10 and expected payoff of reject= 0, thus accept at B3.

B’s strategy consistent with belief:
Reject A if they play and are uninjured. Accept A if they don’t play.

A’s strategy based on the table 2 of expected payoff the best responses of A is:
Not play whether Fit or Unfit

Which is not a separating strategy, and not consistent with the beliefs of B. Thus not a PBNE.

Table 2: Expected Payoffs of A

<table>
<thead>
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<th>Scratch work: Payoff of Play</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit A</td>
<td>-2</td>
<td>0</td>
<td>$0.1 \times (-20) + 0.9 \times 0$</td>
</tr>
<tr>
<td>Unfit A</td>
<td>-10</td>
<td>0</td>
<td>$0.5 \times (-20) + 0.5 \times 0$</td>
</tr>
</tbody>
</table>

In conclusion: a separating PBNE:
B’s posterior beliefs:
$a = 1$ (if Play, always Fit), and
$b = 0$ (if Not, always Unfit)

B’s strategy:
Accept A if Play and are uninjured. Reject A if Not play.

A’s strategy:
If Fit A then Play. If Unfit A then Not .
(c) Pooling strategy:
Player A can play a pooling strategy if B’s belief after seeing the signals is the same as the prior belief:
\[ a = \frac{1}{3} \text{ and } b = \frac{2}{3}. \]

B’s expected payoffs at each information set:
If they are at B1, strategy is to reject,

If they are at B2,
expected payoff if accept = \( \frac{1}{3} \times 10 - \frac{2}{3} \times 10 = -3\frac{1}{3} \), and
expected payoff if reject = 0, thus strategy at B2 is to reject.

If they are at B3,
expected payoff if accept = \( \frac{1}{3} \times 10 - \frac{2}{3} \times 10 = -3\frac{1}{3} \), and
expected payoff if reject = 0, thus strategy at B3 is to reject.

B’s strategy consistent with belief:
Always reject.

BR of A: knowing B’s strategy of always reject, we make the table below, and see that the best response of A is to never take the risk of the dangerous game, and this will be a PBNE.

<table>
<thead>
<tr>
<th></th>
<th>Play</th>
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<td>-5</td>
<td>0</td>
<td>(0.5 \times (-20) + 0.5 \times 10)</td>
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Thus, a pooling PBNE:
B’s posterior beliefs:
\[ a = b = \frac{1}{3} \text{ always a } \frac{1}{3} \text{ chance that A is fit.} \]

B’s strategy:
Always reject A.

A’s strategy:
Whether Fit or Unfit, Not play.
The owner of a new restaurant is planning to advertise to attract customers. In the Bayesian game, Nature determines the restaurant’s quality, which is either high or low. Assume that each quality occurs with equal probability. After the owner learns about quality, he decides how much to advertise. Let $A$ denote the amount of advertising expenditure. For simplicity, assume that there is a single consumer. The consumer observes how much advertising is conducted, updates her beliefs about the quality of the restaurant, and then decides whether or not to go to the restaurant. (One can imagine that $A$ is observed by noticing how many commercial spots are on local television and how many ads are in the newspaper and on billboards.) Assume that the price of a meal is fixed at $50. The value of a high-quality meal to a consumer is $85 and of a low-quality meal is $30. A consumer who goes to the restaurant and finds out that the food is of low quality ends up with a payoff of $−20$, which is the value of a low-quality meal, $30$, less the price paid, $50$. If the food is of high quality, then the consumer receives a value of $35 (= 85 − 50)$. Furthermore, upon learning of the high quality, a consumer anticipates going to the restaurant a second time. Thus, the payoff to a consumer from visiting a high-quality restaurant is actually $70 (= 2 \times 35)$. For the restaurant owner, assume that the cost of providing a meal is $35 whether it is of low or high quality. If the restaurant is of high quality, the consumer goes to the restaurant, and the restaurant spends $A$ in advertising, then its profit (and payoff) is $2 \times (50 − 35) − A = 30 − A$. If the restaurant is of low quality, the consumer goes to the restaurant, and the restaurant spends $A$ in advertising, then its profit is $(50 − 35) − A = 15 − A$. These payoffs are summarized in the following table. If the consumer does not go to the restaurant, then her payoff is zero and the owner’s payoff is $−A$.

<table>
<thead>
<tr>
<th>Restaurant Quality</th>
<th>Owner’s Payoff</th>
<th>Customer’s Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>$15 − A$</td>
<td>$−20$</td>
</tr>
<tr>
<td>High</td>
<td>$30 − A$</td>
<td>$70$</td>
</tr>
</tbody>
</table>

a. Find a separating PBNE.

ANSWER: Consider the following separating strategy profile where $A’ > A$:

**Owner’s strategy:**
- If of high quality, then spend $A’$ on advertising.
- If of low quality, then spend $A$ on advertising.

**Customer’s strategy:**
- If advertising is at least $A’$, then go to the restaurant.
- If advertising is less than $A’$, then do not go to the restaurant.

**Customer’s beliefs:**
- If advertising is at least $A’$, then the restaurant is high quality with probability 1.
- If advertising is less than $A’$, then the restaurant is low quality with probability 1.
Starting with the customer’s beliefs, consistency requires that the customer believe the restaurant is high quality when she observes advertising of $A'$ and is low quality when she observes advertising of $A^\circ$. Thus, these beliefs are consistent. Turning to her strategy, first note that the customer finds it optimal to go to the restaurant if she believes it is high quality—realizing a payoff of 70 versus 0 from not going—and optimal not to go if she believes it is low quality (realizing a payoff of −20 versus 0). According to her beliefs, the restaurant is high quality if advertising is at least $A^\circ$. As her strategy has her go to the restaurant if advertising is at least $A'$, prescribed behavior is optimal. She believes it is of low quality when advertising falls below $A'$; again her strategy is optimal.

Finally, consider the owner’s strategy. If the restaurant is of high quality, the payoff from advertising $A$ is $30 - A$ when $A \geq A'$ and zero when $A < A'$. Hence, advertising $A'$ is optimal if and only if $A' \geq 30$. When the restaurant is of low quality, the payoff from advertising $A$ is $15 - A$ when $A \geq A'$ and zero when $A < A'$. Hence, advertising $A^\circ$, when $A^\circ < A'$, is optimal if and only if $A^\circ = 0$ and $A' \geq 15$. That is, if $A' < 15$, then the restaurant earns a higher payoff (of $15 - A'$) by advertising $A'$ than by advertising $A^\circ$ (which results in a payoff of $-A^\circ$). And if advertising $A^\circ$ means no customers are going to come—so the payoff is $-A^\circ$—then optimality requires $A^\circ = 0$. There is no point in advertising if it doesn’t deliver any customers. In sum, this is a separating perfect Bayes-Nash equilibrium if and only if $A^\circ = 0$ and $15 \leq A' \leq 30$. Note that advertising provides no direct information. Rather it is a signal of a restaurant’s quality. A high-quality restaurant is willing to advertise more to induce a customer to try its food because it knows the customer will return in the future.

b. At a separating PBNE, what is the maximum amount of advertising that a restaurant conducts? What is the minimum amount?

**ANSWER:** With this separating equilibrium, advertising cannot exceed 30 and cannot be less than 15. If it exceeds 30, then a high-quality restaurant would prefer not to advertise at all. If it is less than 15, then the low-quality restaurant would imitate the high-quality restaurant.

c. Find a pooling PBNE.

**ANSWER:** Consider the following separating strategy profile.

**Owner’s strategy:**
If restaurant is of high quality, then spend $A^\circ$ on advertising.
If restaurant is of low quality, then spend $A^\circ$ on advertising.

**Customer’s strategy:**
If advertising is at least $A^\circ$, then go to the restaurant.
If advertising is less than $A^\circ$, then do not go to the restaurant.

**Customer’s beliefs:**
If advertising is at least $A^\circ$, then the restaurant is high quality with probability $\frac{1}{2}$.
If advertising is less than $A^\circ$, then the restaurant is high quality with probability $p$.

Since the customer’s beliefs are the same as her prior beliefs when $A = A^\circ$, beliefs are consistent. Given those beliefs, it is optimal to go to the restaurant when $A \geq A^\circ$ if and only if:

$$\frac{1}{2} \times 70 + \frac{1}{2} \times (-20) \geq 0,$$
where the left-hand expression is the expected payoff from going, which equals 25, and the right-hand expression is the payoff from not going. This inequality holds. It is optimal not to go to the restaurant when $A < A^o$ if and only if

\[ p \times 70 + (1 - p) \times (-20) \leq 0, \]
\[ p \leq \frac{2}{9}. \]

Thus, we must have $p \leq \frac{2}{9}$. For the restaurant, it either wants to advertise $A^o$ (the minimum amount necessary to induce a customer to come) or zero. The former yields a higher expected payoff when:

- High-quality restaurant: $30 - A^o \geq 0$ or $A^o \leq 30$
- Low-quality restaurant: $15 - A^o \geq 0$ or $A^o \leq 15$.

Thus, we need $A^o \leq 15$.

d. At a pooling PBNE, what is the maximum amount of advertising?

**ANSWER:** The maximum amount of advertising at a pooling equilibrium is 15. If it exceeds 15, then the low-quality restaurant owner would prefer to advertise zero.