DUE OCTOBER 18, 2016

PART I Problems – Submit as an INDIVIDUAL

1. Harrington Chapter 5, problem 3, page 171

2. On Thursday October 6, we played a game in class called “minimum effort”. In that game, 5 players each choose how many minutes to shave off their time to compete a sub-task. The payoff of each individual depends on how hard they work (how much they shave from their time) and the minimum effort of the group (since the sub-task that takes the longest governs when the project finishes). Thus the payoff function of player \( i \) is

\[
2 \times \min_{j=1...5} (s_j) - s_i
\]

a) Find all Nash equilibria of this game
b) Compare your result to the data from our play in class. Does it look like play converged to any of the Nash equilibria? If so, which one?

3. Harrington Chapter 6, problem 12, page 212

4. Alice and Bob want to pursue a business venture. They have complementary skills, and the venture needs both of them to put in time to make the venture succeed. In particular the value of the venture is given by the expression

\[
x_a^{1/3} x_b^{1/3}
\]

where \( x_a \) and \( x_b \) are the amounts of time that Alice and Bob put into the venture respectively. Note that this function is concave (grows slower and slower) with respect to each variable. This reflects that there are diminishing returns to effort. Alice and Bob’s costs for investing the time they put in are 2\( x_a \) and \( x_b \) respectively. The factor of two for Alice reflects that her time is more valuable. Suppose that Alice’s share of the venture’s final value is \( s \) and Bob’s is \( 1 - s \). Assume \( 0 < s < 1 \). Thus Alice’s payoff is

\[
s x_a^{1/3} x_b^{1/3} - 2 x_a
\]

and Bob’s payoff is

\[
(1 - s) x_a^{1/3} x_b^{1/3} - x_b
\]

a. What are Alice’s and Bob’s best response functions?
b. Is both players playing 0 a Nash Equilibrium?
c. Find a Nash Equilibrium in which each player chooses a positive amount of time? Express each player’s equilibrium action as a function of \( s \).
d. In the Nash equilibrium found in part c, what is $x_a^{1/3}x_b^{1/3} - 2x_a - x_b$, the combined payoff of Alice and Bob? Express your answer in terms of $s$.

e. Suppose now that Alice and Bob ask you to choose $s$, Alice’s share of the venture, to maximize the Nash equilibrium value of the venture.

Part II PROJECT PROSPECTUS – Submit as a GROUP

1. As a group, submit a project prospectus. It should be no more than three pages long. Please begin by stating the particular question your team will investigate. You may add a paragraph or two of motivation and background. Explain who the players are in the situation of interest, and explain why there is strategic interdependence. Explain your plan to study the situation. Be sure to list the sources of information you expect to use, and to list the members of your team.