Midterm Exam

Instructions. In class, closed book, 105 minutes. Partial credit will be granted for brief, relevant remarks and for partial results, but not for core dumps. Points as marked; total is 50.

1. Consider the two-player game described by the following payoff matrix.

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<tbody>
<tr>
<td>U</td>
<td>68,63</td>
<td>75,65</td>
<td>50,60</td>
</tr>
<tr>
<td>M</td>
<td>65,75</td>
<td>70,70</td>
<td>53,73</td>
</tr>
<tr>
<td>D</td>
<td>60,50</td>
<td>76,53</td>
<td>40,40</td>
</tr>
</tbody>
</table>

a. Does either player have a dominant strategy? (1 point)
b. Does either player have a dominated strategy? (1 point)
c. Which strategy profiles survive iterated deletion of strictly dominated strategies? (2 points)
d. Find all Nash equilibria (NE) of the game. (3 points)

2. Your connection drops during a phone call with your friend. You can either call right back or wait 20 seconds, and so can she; to keep things simple, assume that there are no other options. You both get payoff 0 if you both try to call right back (since the phones are busy), and -1 if you both wait. You get payoff 1 and she gets payoff 2 if you call right back while she waits, and in the reverse case you both get payoff 2.

a. Write the strategic form for this game. (2 pts)
b. Find all pure NE of the game or show that none exist. (3 pts)
c. Find all mixed NE of the game, or show that none exist. (3 pts)
d. Which NE, if any, would you predict as the most likely outcome, and why? (1 pt, max 25 words)

3. Consider the following two player costly waiting game where there is a prize of 100 claimed by the last player remaining, but waiting costs each player 15 in each stage. Player A has the first move: leave or wait. Leaving yields payoffs \((u_A, u_B) = (0, 100)\) and waiting gives Player B the move, again either leave or wait. At this stage, leaving yields payoff vector \((u_A, u_B) = (85, -15)\) and staying gives the move back to Player A. If she leaves at this point the payoffs are (-30, 70), and if she waits then Player B has the final move. Payoffs are (55, -45) if he leaves and are (-10, -10) if he remains, because at this stage each player gets half the prize minus waiting costs of 60 each.

a. Draw the extensive form (game tree). (3pts)
b. Solve the game by backwards induction. (3pts)
c. Write out the strategy sets for both players. (2pts)
d. Write out the strategic form as a bimatrix. (2pts)
e. Find all NE of the game. (3pts)
f. Which NE are subgame perfect? (1pt)

Please turn over the page
4. Only 8 fishermen have access to a lake, and each can fish up to 60 hours per month. Let \( h_i \) be the hours of fisherman \( i \), and let \( H = \sum h_i \) be the fishermen’s total hours. The total catch is \( Y = H(500-H) \). Each fisherman gets his proportionate share \( (h_i/H)Y = h_i(500- H) \) less the cost of his time, equivalent to 50 per hour, so his payoff is

\[
u_i(h_i, H_i) = h_i(500- H_i - h_i) - 50h_i
\]

where \( H_i = H - h_i \) is the other fishermen’s total time.

a. Suppose that other fishermen choose to fish for \( H_\text{1} = 300 \) hours. What is fisherman #1’s best response? (2pts)

b. Find fisherman #1’s best response to an arbitrary \( H_\text{1} \). (3pts)

c. Find a NE to the fishing game. (3pts)

d. If the fishermen could enforce an overall quota on hours fished \( H \), what choice \( H^* \) would maximize total payoff? (2pts)

e. Is there a tragedy of the commons here? Explain briefly (30 words max), and compute the efficiency loss (or, if you prefer, “the price of anarchy”), if any. (3pts)

5. Your business partner in Russia asks for $1 million to invest in a project that definitely will have value $3 million, and promises to return $2 million to you. You fear that he will keep all $3 million himself.

a. Draw the game tree, with yourself as player #1 choosing whether or not to invest, and your partner as player #2 deciding whether or not to repay $2 million. (2pts)

b. Find all subgame perfect Nash equilibria (SPNE) of the game. (2pts)

c. How could including a credible commitment improve the SPNE outcome for both players? (2pts)

d. As a practical matter, what commitment devices can you think of that might work in this situation? (1pt)