Problem 1

(a) There are 6 pure-strategy Nash Equilibria. (1,1,1,1,1,1), (2,2,2,2,2,2), . . . , (6,6,6,6,6,6). For any \( s_i \neq s_j \) it cannot be a NE, since players with larger number will always deviate. When all other players play \( s_{-i} \), there is no reason for player \( i \) to choose a number smaller than \( s_{-i} \). Therefore, all the cases when 6 players have the same number are NE.

(b) In most cases it converges to (1,1,1,1,1). Note that, (6,6,6,6,6) is the Pareto dominant NE, and the empirical NE is the least efficient.

Problem 2

(a) Denote \( X_{-i} = \sum_{j \neq i} x_j \), i.e., the total amount of other players. Given \( X_{-i} \), player \( i \) chooses \( x_i \) to maximize \( (240 - X_{-i} - x_i)x_i \). First-order condition (henceforth F.O.C.) implies that \( 240 - X_{-i} - 2x_i = 0 \), thus \( x_i = \frac{240 - X_{-i}}{2} \) is the best response for player \( i \).

(b) In a symmetric NE, \( x_i = x_j \) and \( X_{-i} = 5x_i \). Solving the best response function and we have \( x_i = \frac{240 - 5x_i}{2} \). Therefore \( x_i^* = 240/7 \approx 34.3 \) is the symmetric NE strategy.

(c) The social welfare is \( 6(240 - 6x_i^*)x_i^* = \frac{345600}{49} \approx 7053.1 \).

(d) The optimal level of social welfare is \( 6(240 - 120)20 = 14400 \). Social efficiency loss is \( (14400 - 7053.1)/14400 = 51.02\% \).

(e) The efficiency loss in actual outcomes is less than NE efficiency loss. Specifically, empirical average fishing time is around 25 (less that NE choice of 34.3 but more than social optimal choice of 20), which leads to an efficiency loss of about 6.3%.

Problem 3

(a) Given \( x_{-i} \), player \( i \) chooses \( x_i \) to maximize \( \sqrt{x_i + x_{-i}} - x_i \). F.O.C. implies that \( \frac{1}{2\sqrt{x_i + x_{-i}}} - 1 = 0 \), that is, \( x_i = \frac{1}{16} - x_{-i} \) is the best response function.

(b) Let \( x_1 = 1/16 - x_2 \) and \( x_2 = 1/16 - x_1 \), we have multiple NE: \( \{(x_1, x_2)|x_1 \geq 0, x_2 \geq 0, x_1 + x_2 = 1/16\} \). Note that the total contribution is constant (1/16).
(c) Take F.O.C. with respect to $T$, we have \( \frac{1}{2} \frac{1}{\sqrt{T}} - 1 = 0 \), which derives $T = \frac{1}{4}$ is the social optimal choice.

(d) F.O.C. implies that \( \frac{1}{4} \frac{1}{\sqrt{x_i + x_{-i} - 10}} - 1 + \frac{10x_{-i}}{(x_i + x_{-i})^2} = 0 \). Add the best response function for player 1 and player 2 together, \( \frac{1}{2} \frac{1}{\sqrt{x_i + x_{-i} - 10}} - 2 + \frac{10}{x_i + x_{-i}} = 0 \). Denote $G = x_1 + x_2 - 10$, then \( \frac{1}{2} \frac{1}{\sqrt{G}} - 2 + \frac{10}{G+10} = 0 \). Solving for $G$ numerically we get the net contribution $G \approx 0.239$. Figure 1 shows the graph of function $G$. 

Figure 1: $G$ Function, showing solution