Part I. Word Problems

Game 1:
PLAYERS: 1, 2. Let choices for both players be denoted “L” or “R” for left or right
P1 Strategy set: {L, R}
P2 Strategy set: {LL, LR, RL, RR}, where xy means “x if player 1 chose L, and y if player 1 chose R.”

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>LR</th>
<th>RL</th>
<th>RR</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 L</td>
<td>1, -2</td>
<td>1, -2</td>
<td>-2,10</td>
<td>-2,10</td>
</tr>
<tr>
<td>P1 R</td>
<td>-3,5</td>
<td>2, -2</td>
<td>-3,5</td>
<td>2, -2</td>
</tr>
</tbody>
</table>

Game 2:
P1 Strategy set: {L, R}
P2 Strategy set: {L, R}

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 L</td>
<td>1, -2</td>
<td>-2,10</td>
</tr>
<tr>
<td>P1 R</td>
<td>-3,5</td>
<td>2, -2</td>
</tr>
</tbody>
</table>

2. No IDSDS solutions for either. One pure NE where P1 plays L and P2 plays RL for game 1 and no pure NE for game 2.

3. 
   a) For \(i \in \{1, 2, \ldots, 5\}\)
      \[u_i = 10^i [x_i = Z] + 3n_{xi}\]
   
   For \(i \in \{6, 7, \ldots, 10\}\)
   \[u_i = 7^i [x_i = Y] + 3n_{xi}\]

Consider the case where each player owns the system that is idiosyncratically most appealing to them, i.e. the strategy profile \((Z, Z, Z, Z, Y, Y, Y, Y)\). The payoff to each of the first 5 players is \(10 + 3(5) = 25\). The payoff they each would get by switching unilaterally to \(Y\) is \(3(6) = 18 < 25\). So they cannot benefit by changing strategies; they all are already making a BR (best response).

The payoff to the last 5 players at the given profile is \(7 + 3(5) = 22\), and deviating to \(X\) would give \(3(6) = 18 < 22\). Again, they are all making a BR. Since everyone is making a BR, the profile \((Z, Z, Z, Z, Y, Y, Y, Y, Y, Y, Y)\) is a NE.

It also follows from the above that neither \((Z, Z, Z, Z, Y, Y, Y, Y, Y)\) nor \((Z, Z, Z, Z, Z, Y, Y, Y, Y, Y)\) can be a NE as we have already calculated the payoffs for these and found that they were worse than \((Z, Z, Z, Z, Y, Y, Y, Y, Y, Y)\) for the player that switched. The same is true for any permutation that leaves constant the numbers of each type of player choosing each of the two strategies.

What about the corners, \((Z, Z, Z, Z, Z, Z, Z, Z)\) and \((Y, Y, Y, Y, Y, Y, Y, Y)\)? Payoff to each player 1-5 for the all-Z profile is \(10 + 10(3) = 40\). Deviating to \(Y\) by any of those players gives \(0 + 1(3) = 3 < 40\), so sticking with \(Z\) is the BR for each of them. Similarly for players players 6-10: sticking with \(Z\) gives payoff \(0 + 10(3) = 30\), while switching gives \(7 + 1(3) = 10 < 30\). Therefore, all-Z is a NE. A parallel argument shows that all-Y is also a NE.

All that is left is ruling out other configurations. The above already showed that we cannot have an NE with 9
people playing one console and 1 person playing the other. Now consider the situation where everybody plays Z except two people from the Y-preferring group 6-10. Those two can benefit from switching to Z and getting $0+3(8) = 24$ instead of the $7+3(2) = 13$ that they are currently getting, so this cannot be a NE. Perform this same exercise for 7 people playing Z and 3 people playing Y and you will find the same. As we have already excluded the possibility of 6 people playing one and 4 playing the other in analyzing the equilibrium with everybody playing the idiosyncratically preferred console, we are almost done. Check the 5 – 5 split with everyone owning the less idiosyncratically appealing console (Y,Y,Y,Y,Z,Z,Z,Z): every player can profitably deviate!

So there are exactly 3 pure NEs: all-Y, all-Z and the natural 5-5 split (Z,Z,Z,Z,Z,Y,Y,Y,Y).

b) None of them are payoff dominant.

c) The game has tipping.

4. (a)

(b) The strategy sets for Prof is \{M,W,F\}, and if we eliminate the dominated strategies of ever choosing L on Friday,* then the strategy set for Student has $2^3=8$ elements of the form xyz, meaning choose x (=T or L) at the first info set (for Monday), y at the second (for Wednesday if chose T on Monday) and z at the third (for Wednesday if chose L on Monday).

*If we don’t eliminate those dominated strategies, then there are $2^7=128$ strategies for the student! We can further reduce the Student’s strategies by noting that the payoff from Wednesday’s guess is independent of previous guesses, so there is no reason for z to differ from y.

Therefore, the REDUCED strategy set for Student would be \{TT,TL,LT,LL\}, e.g., where TT implies “Guess Today on Monday, and if it comes to Wednesday, then guess Today”. (Whatever happens, on Friday student will always guess today.) Then the reduced game bimatrix is
(c) There is no pure strategy Nash Equilibrium. But later we will see that there is a mixed NE.

### Part II. Textbook Problems

#### 3.3

3. For the Team-project game, suppose a jock is matched up with a sorority girl as shown in Figure PR3.3.

![Figure PR3.3](image)

<table>
<thead>
<tr>
<th>Sorority girl</th>
<th>Low</th>
<th>Moderate</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>3.0</td>
<td>4.1</td>
<td>5.2</td>
</tr>
<tr>
<td>Moderate</td>
<td>2.2</td>
<td>3.4</td>
<td>4.3</td>
</tr>
<tr>
<td>High</td>
<td>1.6</td>
<td>2.5</td>
<td>3.4</td>
</tr>
</tbody>
</table>

**a.** Assume that both are rational and that the jock knows that the sorority girl is rational. What happens?

**Answer:** For the jock, both low and moderate strictly dominate high, and low strictly dominates moderate. None of the sorority girl's strategies is strictly dominated, however. After eliminating the strictly dominated strategies, the reduced game is as shown in Figure SOL3.3.1. As we don't know what the sorority girl believes about the jock, we cannot go any further. The answer is that the jock chooses low and the sorority girl chooses low, moderate, or high.

![Figure SOL3.3.1](image)

<table>
<thead>
<tr>
<th>Sorority girl</th>
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</table>

**b.** Assume that both are rational and that the sorority girl knows that the jock is rational. What happens?

**Answer:** With the game shown in Figure SOL3.3.1, the sorority girl now knows the jock is rational and thus will play low. Hence, she should choose high as it strictly dominates both low and moderate. Hence, the jock chooses low effort and the sorority girl chooses high effort.

#### 3.5

1\textsuperscript{st} round of deletion: No strategies deleted for player 1, x strictly dominates z for player 2

2\textsuperscript{nd} round of deletion: In reduced game, a,b,c all strictly dominate d for player 1, no strategies deleted for player 2

After this, no further strategies can be eliminated, so remaining (rationalizable) strategies are a,b,c for P1 and x,y for P2
3.18
1st round of deletion: for player 3, a3 strictly dominates c3
2nd round of deletion: in reduced game, for player 2, b2 strictly dominates c2
3rd round of deletion: in reduced game, for player 3, a3 strictly dominates b3
4th round of deletion: in reduced game, for player 1, c1 strictly dominates a1
5th round of deletion: in reduced game, for player 2, a2 strictly dominates b2
final round of deletion: in reduced game, for player 1, c1 strictly dominates b1.
Therefore, only the strategy profile (c1,a2,a3) survives IDDS.

4.3

\[
\begin{array}{c|ccc}
\text{Sorority girl} & \text{Low} & \text{Moderate} & \text{High} \\
\hline
\text{Low} & 0,0 & 2,1 & 6,2 \\
\text{Moderate} & 1,2 & 4,4 & 5,3 \\
\text{High} & 2,6 & 3,5 & 3,4 \\
\end{array}
\]

**ANSWER:** There is one symmetric Nash equilibrium, (moderate, moderate), and two asymmetric Nash equilibria, (low, high) and (high, low).

4.11

**ANSWER:** After using the best-reply method (where a best reply is denoted by an asterisk), we have Figure SOL4.12.1

**FIGURE SOL4.12.1**

The Nash equilibria are (C/C, torture, refer), (C/DNC, torture, refer), (DNC/C, DNT, do not refer), (DNC/DNC, torture, do not refer) and (DNC/DNC, DNT, do not refer).