4.8 L’Hospital’s rule: limits revisited

Textbook pages 299-307

L’Hospital’s rule is a very powerful tool for finding limits of indeterminate limits such as

**Rule for the indeterminate form 0/0:**

**Examples for the indeterminate form 0/0:**

- Find the limit of \( \frac{\sin(x)}{x} \) at \( x = 0 \).

- Find the limit of \( \frac{x^2 - 9}{x - 3} \) at \( x = 3 \).

- Find the limit of \( \frac{\tan(x)}{x} \) at \( x = 0 \).
Proof for the indeterminate form $0 \div 0$:

Example for the indeterminate form $\infty \div \infty$:

- Find $\lim_{x \to \infty} \frac{\ln(x)}{x}$.

- Find $\lim_{x \to \frac{\pi}{2}} \frac{\tan(x)}{1 + \tan(x)}$.

The proof of this indeterminate form is more difficult, so we will simply assume that it is true.

Advantages:
- L'Hospital Rule can be applied multiple times
L'Hospital Rule can also be used to find limits of other indeterminate forms provided we can cast them into either of the above $0/0$ or $\infty/\infty$.

**Examples:**

- Find $\lim_{x \to 0} \frac{x^2}{\sin^2(x)}$.

- Find $\lim_{x \to 0} \left( \frac{1}{\sin(x)} - \cot(x) \right)$.

**IMPORTANT NOTE:** L'Hospital rule ONLY works for these two indeterminate forms. Do not try to use it without verifying that the limit is one of those forms first. For example, you cannot use L'Hospital Rule on the following problems:

- $\lim_{x \to 3} \frac{x^2 - 1}{x - 3}$
- $\lim_{x \to 0} \frac{x}{\ln(x)}$
- $\lim_{x \to \infty} \frac{e^{-x}}{x^2}$

(and so forth)...
Check your understanding of Lecture 22

- **Limits using L’Hospital’s Rule**
  Do as many problems as you can from the following list: Textbook page 307 numbers 1 through 32 (a good start would be all of the odd-numbered, or all of the even numbered problems on that list).

- **Further examples of L’Hospital’s Rule**
  Textbook page 308 number 51, 56.