1. Suppose you run a retail store that sells widgets. During any day, your customers try to buy one widget with probability 0.3, two widgets with probability 0.1 and no widgets with probability 0.6. A sale is successful only if you have enough widgets in stock. (If you had the opportunity to sell 2 widgets, but you had only 1 in stock that day, you just sell 1.) Suppose that you pay an inventory cost of $2 for every widget you hold in stock overnight, and you earn $10 for each widget sold. Suppose that at the end of each business day, your control decision is to order 0, 1 or 2 widgets. There is a delay in receiving the ordered widgets. In particular, a widget that is ordered at the end of the day \( i \), is available for sale on the morning of day \( i + 2 \).

(a) Suppose your control policy is to order 2 whenever your stock at the end of a day is 0; otherwise you order 0. What is your expected long term average profit? Hint: Model your state as a Markov Chain with states labeled \((X_i, Y_i)\) where \(X_i, Y_i \in \{0, 1, 2\}\) and \(X_i\) and \(Y_i\) are defined as described in the next part. This would give you a 9 state Markov chain, but you should eliminate any unreachable states to simplify your analysis.

(b) Suppose \(X_i\) is your stock at the end of day \( i \), and \(Y_i\) is the amount you ordered on the evening of day \( (i - 1) \) and that you expect to arrive on the morning of day \( (i + 1) \). Suppose your control policy is to order \(2 - (X_i + Y_i)\). What is your long term average profit? You may use a computer to help you with the computations. You will have to find \(\pi\) to solve \(\pi M = \pi\) where \(M\) is a \(n\) by \(n\) transition matrix. Matlab, which is available to you in the computer lab, can solve this easily. You should use the following steps:

- Suppose \(m_{ij}\) is the value of the \((i, j)\) entry of your matrix \(M\), enter into Matlab:

\[
M = \begin{bmatrix}
m_{11}, m_{12}, ..., m_{1n}; & m_{21}, m_{22}, ..., m_{2n}; & \cdots; & m_{31}, m_{32}, ..., m_{nn}
\end{bmatrix}
\]

For example, if

\[
M = \begin{bmatrix}
.3 & .7 \\
.8 & .2
\end{bmatrix}
\]

enter \(M = [.3, .7; .8, .2]\)

- Now enter

\[
[V, D] = eig(M')
\]

This will find the right eigenvectors and eigenvalues of \(M^T\), which of course are the same as the left eigenvectors of \(M\).

- Matlab will display \(V\), a matrix whose columns are eigenvectors, and \(D\), a diagonal matrix whose diagonal entries are eigenvalues. Look for the entry of \(D\) that has value 1. Say it is entry \((j, j)\). Then the \(j\)th column of \(V\) is the vector we want. The \(j\)th column of \(V\) can be extracted by typing \(V(:, j)\), To normalize the entries of \(V(:, j)\) so that they sum to one type this command:

\[
V(:, j)/\text{sum}(V(:, j))
\]

(c) Which control policy is best? Can you select a different inventory cost (different than the $2 we assumed) that would lead to a different one of the two control policies being the best?

2. Durrett 9.28 pp 93
3. Durrett 9.38 pp 96
4. Durrett 9.41 pp 97
5. Durrett 9.48 pp 98