1. Durrett 8.46 pp 207

2. Suppose you manage a network consisting of 3 stations, and each station is a M/M/1 queue. Customers arrive to the network at rate \( \lambda \), and all new arrivals to the network arrive to station 1. Customers who finish service at station 1 go to station 2 with probability \( p \) and go to station 3 with probability \( 1 - p \). Customers who finish service at station 2 return to station 1 with probability \( q \) and exit the system with probability \( 1 - q \). Customers who finish service at station 3 always exit the system. The service rates at the stations 1, 2, and 3 are \( \mu_1 \), \( \mu_2 \), and \( \mu_3 \) respectively. (i) Under what conditions will the network be stable? (ii) What is the invariant distribution? (ii) Suppose each unit of service rate costs \$1 \) and you have \$C \) to spend. (Thus your constraint is \( \mu_1 + \mu_2 + \mu_3 \leq C \)). What choice of service rates minimizes the average number of customers queued in the system in steady state.

3. Suppose you manage a call center with Poisson arrivals of rate 3 per minute. The call center is divided into two stations: station 1 has \( s_1 \) “screener” operators and station 2 has \( s_2 \) “expert” operators. The service time of a call in each station is exponentially distributed with a mean of 1 minute. Thus, each station is an M/M/1 queue, and each station has an unlimited “waiting room” for calls to queue while waiting for service. All calls arrive first to station 1. After finishing service at station 1, the calls are either forwarded to station 2 or exit the call center with equal probability. You can think of the calls that get passed from station 1 to station 2 as difficult calls that the screeners in station 1 forwarded to the experts of station 2. Calls that finish service at station 2 always exit the system. (i) What conditions on \( s_1 \) and \( s_2 \) are required for stability? (ii) What is the invariant distribution of this system? (iii) What is the expected number of customers in the system in steady state? It is ok if your expression for this is not simplified. (iv) Suppose that the staff costs of your center are \( s_1 + 2s_2 \) and you must keep your costs at or below \$11. Pick \( s_1 \) and \( s_2 \) to minimize the average number of customers queued in the system in steady state, subject to your budget constraint.

Hints: Argue that the only allocations that you need to compare are \( (s_1, s_2) = (5, 3) \) and \( (7, 2) \). You might want to use matlab or excel to help with the calculations.

4. Durrett 5.16 pp 236

5. Durrett 5.18 pp 237