1. (a) Consider the LTI filter described by the differential equation
\[
\frac{d^2}{dt^2} Y(t) = - (\lambda_1 + \lambda_2) \frac{d}{dt} Y(t) - \lambda_1 \lambda_2 Y(t) + X(t)
\]
where \(\lambda_1 > 0\) and \(\lambda_2 > 0\). Compute the impulse response \(h(t)\) when the system starts from rest.
(i.e. Assuming that \(Y(t) = 0\) \(\forall t \in (-\infty, 0-)\). and finding the \(Y(t)\) that satisfies the differential equation when \(X(t) = \delta(t)\), the Dirac delta function. Recall that the Dirac delta function satisfies \(\delta(t) = 0\) for all \(t \neq 0\) and \(\int \delta(t) dt = 1\).

Hint: Consider functions of the form \(Y(t) = Ae^{-\lambda_1 t} 1(t > 0) + Be^{-\lambda_2 t} 1(t > 0)\). Find the numbers \(A\) and \(B\) so that \(Y(t)\) satisfies the differential equation and the boundary conditions: \(\dot{Y}(0+) = 1\) and \(Y(0+) = 0\).

(b) Find \(H(\omega)\).
(c) Plot \(|H(\omega)|\) as a function of \(\omega\) when \(\lambda_1 = 1\) and \(\lambda_2 = 2\).
(d) Plot \(|H(\omega)|\) as a function of \(\omega\) when \(\lambda_1 = 10\) and \(\lambda_2 = 20\).
(e) Suppose that the input of the filter you have analyzed above is fed white gaussian noise \(X(t) \equiv W(t)\) with p.s.d. \(S_W(\omega) = N_0/2\). What is \(E[Y^2(t)]\) in terms of the variables \(\lambda_1\) and \(\lambda_2\)?

(f) Does increasing \(\lambda_1\) and \(\lambda_2\) tend to increase or decrease the energy of the signal that exits the filter?

2. Durrett Exercise 9.1 pp 88
3. Durrett Exercise 9.3 pp 89
4. Durrett Exercise 9.5 pp 89
5. Durrett Exercise 9.7 pp 89
6. Durrett Exercise 9.9 pp 90