1. (a) Suppose that $Y$ is J.G. with zero mean and covariance $K_Y = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

Find a matrix $A$ such that $W := A^{-1}Y$ satisfies $W \sim N(0, I)$.

(b) What is the probability that $1y_1 + 2y_2 > 4$?

2. Consider the matrix $K_Y = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Find a matrix $A$ and a JG random vector $Z$ such that the covariance matrix of $Y = AZ$ is $K_Y$ and $\det(K_Z) \neq 0$.

3. Let $H(\omega) = \begin{cases} 1 & \text{for } |\omega| \in [\omega_0 - \epsilon/2, \omega_0 + \epsilon/2] \\ 0 & \text{otherwise} \end{cases}$

Show that the inverse Fourier transform of $H(\omega)$ is $h(t) = 2 \cos(\omega_0 t) \frac{\sin(\epsilon t/2)}{\pi t}$.

4. (a) Suppose that $W(t)$ is White Gaussian Noise with p.s.d. $S_W(\omega) = N_0/2$. Suppose that $d(t)$ is a deterministic function satisfying $\int_{-\infty}^{\infty} |d(t)|^2 dt = D$. Let $Z = \int_{-\infty}^{\infty} d(t)W(t)dt$.

What is the distribution of $Z$?

(b) Suppose that your friend sends a 1 represented by the signal $X(t) \equiv d(t)$ with probability 0.5 or sends a 0 represented by the signal $X(t) \equiv 0$ with probability 0.5. In either case, you receive the signal $Y(t) = X(t) + W(t)$. Suppose you decide to build a detector. Your detector starts by computing the random variable $Z := \int_{-\infty}^{\infty} d(t)Y(t)dt$. What is the distribution of $Z$ conditioned on the event that your friend sends a 1? What is the distribution of $Z$ conditioned on the event that your friend sends a 0?

(c) Write the MAP detector, based on the observation $Z$. Use the results from your last homework assignment here.

(d) What is the probability of making an error if $N_0/2 = 1$ and $D = 2$?

(e) Suppose now that you build your detector differently. Instead you compute $Z' := \int_{-\infty}^{\infty} d'(t)Y(t)dt$, where $d'(t)$ is a deterministic signal that is not equal to $d(t)$ and satisfies $\int_{-\infty}^{\infty} |d'(t)|^2 dt = D$ and $\int_{-\infty}^{\infty} d'(t)d(t)dt = D/2$.

What is the distribution of $Z'$ conditioned on the event that your friend sends a 1? What is the distribution of $Z'$ conditioned on the event that your friend sends a 0?

(f) Write the MAP detector, based on the observation $Z'$.

(g) What is the probability of making an error if $N_0/2 = 1$ and $D = 2$?

(h) Which of the two detectors is better?