Midterm Sample Questions
True or counterexample: State whether the following is always true, or if not, find a counterexample to the statement:

1. The sum of two Gaussian Random Variables is always Gaussian.
2. \( E[XY] = E[X] E[Y] \).
3. Two Jointly Gaussian Random vectors with the same mean and covariance matrix have the same distribution (same cumulative distribution functions and joint densities).
4. A Markov chain always converges to a stationary distribution.

Answer the following:

5. \( X \) is JG with covariance
   \[
   \begin{bmatrix}
   6 & 2 \\
   2 & 6
   \end{bmatrix},
   \]
   What is the variance of \( Y = [1 \ 1] X \) ?
6. A linear system has impulse response \( H(t) = e^{-t}1(t > 0) \). Suppose the input is white noise with PSD \( N_0/2 \) for all \( \omega \). Call the output process \( Y(t) \). What is the variance of \( Y(0) \)?
7. A Markov chain \( X \) has two states: A, B. When in A it has a 30% chance of going to B in each period. When in B it has a 60% chance of going to A in each period. What is the limiting fraction of time that it is in B? By limiting fraction we mean,
   \[
   \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} 1(X_n = B).
   \]