Homework 1
ISM 206, Fall 2010
Due: Tuesday, October 12, in class.

Reading: Chapters 1-5 Hillier and Lieberman.
Problems identified by number are taken from Hillier and Lieberman 8th edition. See instructor if you have difficulty accessing the problems.

1. Problem 3.1-9

2. Problem 3.4-14

3. Problem 3.6-3

4. Convert the following linear program into standard form:

\[
\begin{align*}
\text{min} \quad & -7x_2 + 4x_3 - 3x_4 \\
\text{subject to} \quad & 6x_2 \leq 12 \\
& 2x_2 + 4x_3 + x_4 = 10 \\
& x_1 - x_2 + 2x_3 \geq 14 \\
& x_1, x_3 \geq 0, x_2, x_4 \text{ free}
\end{align*}
\]  \tag{1}

5. Problem 4.3-4

6. Show that the solutions to a linear programming problem always form a convex set.

7. Problem 4.6-11
3.1-7. The WorldLight Company produces two light fixtures (products 1 and 2) that require both metal frame parts and electrical components. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of $1, and each unit of product 2, up to 60 units, gives a profit of $2. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.

(a) Formulate a linear programming model for this problem.

(b) Use the graphical method to solve this model. What is the resulting total profit?

3.1-8. The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is $5 per unit on special risk insurance and $2 per unit on mortgages.

Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

<table>
<thead>
<tr>
<th>Department</th>
<th>Work-Hours per Unit</th>
<th>Available Work-Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Special Risk</td>
<td>Mortgage</td>
</tr>
<tr>
<td>Underwriting</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Administration</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Claims</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Formulate a linear programming model for this problem.

(b) Use the graphical method to solve this model.

(c) Verify the exact value of your optimal solution from part (b) by solving algebraically for the simultaneous solution of the relevant two equations.

3.1-9. Weenies and Buns is a food processing plant which manufactures hot dogs and hot dog buns. They grind their own flour for the hot dog buns at a maximum rate of 200 pounds per week. Each hot dog bun requires 0.1 pound of flour. They currently have a contract with Pigland, Inc., which specifies that a delivery of 800 pounds of pork product is delivered every Monday. Each hot dog requires 1 pound of pork product. All the other ingredients in the hot dogs and hot dog buns are in plentiful supply. Finally, the labor force at Weenies and Buns consists of 5 employees working full time (40 hours per week each). Each hot dog requires 3 minutes of labor, and each hot dog bun requires 2 minutes of labor. Each hot dog yields a profit of $0.20, and each bun yields a profit of $0.10.

Weenies and Buns would like to know how many hot dogs and how many hot dog buns they should produce each week so as to achieve the highest possible profit.

(a) Formulate a linear programming model for this problem.

(b) Use the graphical method to solve this model.

3.1-10. The Omega Manufacturing Company has decided to produce a certain unprofitable product line. To raise revenue, it is considering producing one or more of the products; call them products 1, 2, and 3. The available production capacity for the machines that might limit output is summarized in the following table:

<table>
<thead>
<tr>
<th>Machine Type</th>
<th>Available Time (Machine Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milling</td>
<td>500</td>
</tr>
<tr>
<td>Lathe</td>
<td>350</td>
</tr>
<tr>
<td>Grinder</td>
<td>150</td>
</tr>
</tbody>
</table>

The number of machine hours required for each unit of the products is

Productivity coefficient (in machine hours per unit)

<table>
<thead>
<tr>
<th>Machine Type</th>
<th>Product 1</th>
<th>Product 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milling</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Lathe</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Grinder</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

The sales department indicates that the sales potential for product 1 is 20 units per week, for product 2 is 10 units per week, and for product 3 is 5 units per week. The objective is to determine how much of each product Omega should produce to maximize profit.

(a) Formulate a linear programming model for this problem.

(b) Use a computer to solve this model by the simple

3.1-11. Consider the following problem, where the \( c \) has not yet beenascertained.

Maximize \( Z = c_1x_1 + x_2 \),

subject to

\[
\begin{align*}
x_1 + x_2 & \leq 6 \\
x_1 + 2x_2 & \leq 10
\end{align*}
\]

and

\[
\begin{align*}
x_1 & \geq 0, \\
x_2 & \geq 0.
\end{align*}
\]

Use graphical analysis to determine the optimal solution \((x_1, x_2)\) for the various possible values of \(c_1(c_1 < c) < \infty)\).

3.1-12. Consider the following problem, where the \( c \) has not yet been ascertained.

Maximize \( Z = x_1 + 2x_2 \).
PROBLEMS

Any portion of these cargoes can be accepted. The objective is to determine how much (if any) of each cargo should be accepted and how to distribute each among the compartments to maximize the total profit for the flight.

(a) Formulate a linear programming model for this problem.
(b) Solve this model by the simplex method to find one of its multiple optimal solutions.

3.4-14. Oxbridge University maintains a powerful mainframe computer for research use by its faculty, Ph.D. students, and research associates. During all working hours, an operator must be available to operate and maintain the computer, as well as to perform some programming services. Beryl Ingram, the director of the computer facility, oversees the operation.

It is now the beginning of the fall semester, and Beryl is confronted with the problem of assigning different working hours to her operators. Because all the operators are currently enrolled in the university, they are available to work only a limited number of hours each day, as shown in the table below.

<table>
<thead>
<tr>
<th>Food Item</th>
<th>Calories from Fat</th>
<th>Total Calories</th>
<th>Vitamin C (mg)</th>
<th>Protein (g)</th>
<th>Cost (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread (1 slice)</td>
<td>10</td>
<td>70</td>
<td>0</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Peanut butter (1 tbsp)</td>
<td>75</td>
<td>100</td>
<td>0</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Strawberry jelly (1 tbsp)</td>
<td>0</td>
<td>50</td>
<td>3</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Graham cracker (1 cracker)</td>
<td>20</td>
<td>60</td>
<td>0</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Milk (1 cup)</td>
<td>70</td>
<td>150</td>
<td>0</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Juice (1 cup)</td>
<td>0</td>
<td>100</td>
<td>2</td>
<td>1</td>
<td>35</td>
</tr>
</tbody>
</table>

The nutritional requirements are as follows. Each child should receive between 400 and 600 calories. No more than 30 percent of the total calories should come from fat. Each child should consume at least 60 milligrams (mg) of vitamin C and 12 grams (g) of protein. Furthermore, for practical reasons, each child needs exactly 2 slices of bread (to make the sandwich), at least twice as much peanut butter as jelly, and at least 1 cup of liquid (milk and/or juice).

Joyce and Marvin would like to select the food choices for each child which minimize cost while meeting the above requirements.

(a) Formulate a linear programming model for this problem.
(b) Solve this model by the simplex method.

3.5-1. Read the article footnoted in Sec. 3.5 that describes the first case study presented in that section: "Choosing the Product Mix at Ponderosa Industrial."

(a) Describe the two factors which, according to the article, often hinder the use of optimization models by managers.
(b) Section 3.5 indicates without elaboration that using linear programming at Ponderosa "led to a dramatic shift in the types of plywood products emphasized by the company." Identify this shift.
(c) With the success of this application, management then was eager to use optimization for other problems as well. Identify these other problems.
(d) Photocopy the two pages of appendices that give the mathematical formulation of the problem and the structure of the linear programming model.

3.5-2. Read the article footnoted in Sec. 3.5 that describes the second case study presented in that section: "Personnel Scheduling at United Airlines."

(a) Describe how United Airlines prepared shift schedules at airports and reservations offices prior to this OR study.
(b) When this study began, the problem definition phase defined five specific project requirements. Identify these project requirements.
(c) At the end of the presentation of the corresponding example in Sec. 3.4 (personnel scheduling at Union Airways), we pointed out that the divisibility assumption does not hold for
this kind of application. An integer solution is needed, but linear programming may provide an optimal solution that is noninteger. How does United Airlines deal with this problem?

(d) Describe the flexibility built into the scheduling system to satisfy the group culture at each office. Why was this flexibility needed?

(e) Briefly describe the tangible and intangible benefits that resulted from the study.

3.5-3. Read the 1986 article footnoted in Sec. 2.1 that describes the third case study presented in Sec. 3.5: “Planning Supply, Distribution, and Marketing at Citgo Petroleum Corporation.”

(a) What happened during the years preceding this OR study that made it vastly more important to control the amount of capital tied up in inventory?

(b) What geographical area is spanned by Citgo’s distribution network of pipelines, tankers, and barges? Where do they market their products?

(c) What time periods are included in the model?

(d) Which computer did Citgo use to solve the model? What were typical run times?

(e) Who are the four types of model users? How does each one use the model?

(f) List the major types of reports generated by the SDM system.

(g) What were the major implementation challenges for this study?

(h) List the direct and indirect benefits that were realized from this study.

3.6-1. You are given the following data for a linear programming problem where the objective is to maximize the profit from allocating three resources to two nonnegative activities.

<table>
<thead>
<tr>
<th>Resource</th>
<th>Resource Usage per Unit of Each Activity</th>
<th>Amount of Resource Available</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Activity 1</td>
<td>Activity 2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Contribution per unit</td>
<td>$20</td>
<td>$30</td>
</tr>
</tbody>
</table>

(a) Formulate a linear programming model for this problem.

(b) Use the graphical method to solve this model.

(c) Display the model on an Excel spreadsheet.

(d) Use the spreadsheet to check the following solutions: \((x_1, x_2) = (2, 2), (3, 3), (2, 4), (4, 2), (3, 4), (4, 3)\). Which of these solutions are feasible? Which of these feasible solutions has the best value of the objective function?

(c) Use the Excel Solver to solve the model by the simplex method.

3.6-2. Ed Butler is the production manager for the Bileco Corporation, which produces three types of spare parts for automobiles.

<table>
<thead>
<tr>
<th>Part</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine 1</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Machine 2</td>
<td>0.05</td>
<td>0.02</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Each machine is available 40 hours per month. Each part manufactured will yield a unit profit as follows:

<table>
<thead>
<tr>
<th>Part</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>$50</td>
<td>$40</td>
<td>$30</td>
</tr>
</tbody>
</table>

Ed wants to determine the mix of spare parts to produce in order to maximize total profit.

(a) Formulate a linear programming model for this problem.

(b) Display the model on an Excel spreadsheet.

(c) Make three guesses of your own choosing for the optimal solution. Use the spreadsheet to check each one for feasibility and, if feasible, to find the value of the objective function. Which feasible guess has the best objective function value?

(d) Use the Excel Solver to solve the model by the simplex method.

3.6-3. You are given the following data for a linear programming problem where the objective is to minimize the cost of conducting two nonnegative activities so as to achieve three benefits that do not fall below their minimum levels.

<table>
<thead>
<tr>
<th>Benefit</th>
<th>Benefit Contribution per Unit of Each Activity</th>
<th>Minimum Acceptable Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Activity 1</td>
<td>Activity 2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

| Unit cost | $60 | $50 |

(a) Formulate a linear programming model for this problem.

(b) Use the graphical method to solve this model.

(c) Display the model on an Excel spreadsheet.

(d) Use the spreadsheet to check the following solutions: \((x_1, x_2) = (7, 7), (7, 8), (8, 7), (8, 8), (8, 9), (9, 8)\). Which of these solutions are feasible? Which of these feasible solutions has the best value of the objective function?

(e) Use the Excel Solver to solve this model by the simplex method.

3.6-4. Fred Jonsson manages a family-owned farm. To supplement several food products grown on the farm, Fred also raises pigs for market. He now wishes to determine the quantities of the...
4.1.6. Describe graphically what the simplex method does step by step to solve the following problem.

Maximize \( Z = 2x_1 + 3x_2 \),

subject to

\[-3x_1 + x_2 \leq 1 \]
\[4x_1 + 2x_2 \leq 20 \]
\[4x_1 - x_2 \leq 10 \]
\[-x_1 + 2x_2 \leq 5 \]

and

\( x_1 \geq 0, \quad x_2 \geq 0 \).

4.1.7. Describe graphically what the simplex method does step by step to solve the following problem.

Minimize \( Z = 5x_1 + 7x_2 \),

subject to

\[2x_1 + 3x_2 \geq 42 \]
\[3x_1 + 4x_2 \geq 60 \]
\[x_1 + x_2 \geq 18 \]

and

\( x_1 \geq 0, \quad x_2 \geq 0 \).

4.1.8. Label each of the following statements about linear programming problems as true or false, and then justify your answer.

(a) For minimization problems, if the objective function evaluated at a CPF solution is no larger than its value at every adjacent CPF solution, then that solution is optimal.

(b) Only CPF solutions can be optimal, so the number of optimal solutions cannot exceed the number of CPF solutions.

(c) If multiple optimal solutions exist, then an optimal CPF solution may have an adjacent CPF solution that also is optimal (the same value of \( Z \)).

4.1.9. The following statements give inaccurate paraphrases of the six solution concepts presented in Sec. 4.1. In each case, explain what is wrong with the statement.

(a) The best CPF solution always is an optimal solution.

(b) An iteration of the simplex method checks whether the current CPF solution is optimal and, if not, moves to a new CPF solution.

(c) Although any CPF solution can be chosen to be the initial CPF solution, the simplex method always chooses the origin.

(d) When the simplex method is ready to choose a new CPF solution to move to from the current CPF solution, it only considers adjacent CPF solutions because one of them is likely to be an optimal solution.

(e) To choose the new CPF solution to move to from the current CPF solution, the simplex method identifies all the adjacent CPF solutions and determines which one gives the largest rate of improvement in the value of the objective function.

4.2.1. Reconsider the model in Prob. 4.1.4.

(a) Introduce slack variables in order to write the functional constraints in augmented form.

(b) For each CPF solution, identify the corresponding BF solution by calculating the values of the slack variables. For each BF solution, use the values of the variables to identify the nonbasic variables and the basic variables.

(c) For each BF solution, demonstrate (by plugging in the solution) that, after the nonbasic variables are set equal to zero, this BF solution also is the simultaneous solution of the system of equations obtained in part (a).

4.2.2. Reconsider the model in Prob. 4.1.5. Follow the instructions of Prob. 4.2.1 for parts (a), (b), and (c).

(d) Repeat part (b) for the corner-point infeasible solutions and the corresponding basic infeasible solutions.

(e) Repeat part (c) for the basic infeasible solutions.

4.3.1. Work through the simplex method (in algebraic form) step by step to solve the model in Prob. 4.1.4.

4.3.2. Reconsider the model in Prob. 4.1.5.

(a) Work through the simplex method (in algebraic form) by hand to solve this model.

(b) Repeat part (a) with the corresponding interactive routine in your OR Tutor.

(c) Verify the optimal solution you obtained by using a software package based on the simplex method.

4.3.3. Work through the simplex method (in algebraic form) step by step to solve the following problem.

Maximize \( Z = 4x_1 + 3x_2 + 6x_3 \),

subject to

\[3x_1 + x_2 + 3x_3 \leq 30 \]
\[2x_1 + 2x_2 + 3x_3 \leq 40 \]

and

\( x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \).

4.3.4. Work through the simplex method (in algebraic form) step by step to solve the following problem.

Maximize \( Z = x_1 + 2x_2 + 4x_3 \),

subject to

\[3x_1 + x_2 + 5x_3 \leq 10 \]
\[x_1 + 3x_2 + x_3 \leq 8 \]
\[2x_1 + 2x_2 \leq 7 \]

and

\( x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \).

4.3.5. Consider the following problem.

Maximize \( Z = 5x_1 + 3x_2 + 4x_3 \),

subject to

\[2x_1 + x_2 + x_3 \leq 20 \]
\[3x_1 + x_2 + 2x_3 \leq 30 \]
c (b) Use a computer package based on the simplex method to determine that the problem has no feasible solutions.

1 (c) Using the Big M method, work through the simplex method step by step to demonstrate that the problem has no feasible solutions.

1 (d) Repeat part (c) when using phase 1 of the two-phase method.

4.6-6. Follow the instructions of Prob. 4.6-5 for the following problem.

Minimize \[ Z = 5,000x_1 + 7,000x_2, \]
subject to
\[-2x_1 + x_3 \geq 1 \]
\[x_1 - 2x_2 \geq 1 \]

and
\[x_1 \geq 0, \quad x_2 \geq 0.\]

4.6-7. Consider the following problem.

Maximize \[ Z = 2x_1 + 5x_2 + 3x_3, \]
subject to
\[x_1 - 2x_2 + x_3 \geq 20 \]
\[2x_1 + 4x_2 + x_3 = 50 \]

and
\[x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.\]

(a) Using the Big M method, construct the complete first simplex tableau for the simplex method and identify the corresponding initial (artificial) BF solution. Also identify the initial entering basic variable and the leaving basic variable.

1 (b) Work through the simplex method step by step to solve the problem.

1 (c) Using the two-phase method, construct the complete first simplex tableau for phase 1 and identify the corresponding initial (artificial) BF solution. Also identify the initial entering basic variable and the leaving basic variable.

1 (d) Work through phase 1 step by step.

(e) Construct the complete first simplex tableau for phase 2.

1 (f) Work through phase 2 step by step to solve the problem.

(g) Compare the sequence of BF solutions obtained in part (b) with that in parts (c) and (f). Which of these solutions are feasible only for the artificial problem obtained by introducing artificial variables and which are actually feasible for the real problem?

C (h) Use a software package based on the simplex method to solve the problem.

4.6-8. Consider the following problem.

Minimize \[ Z = 2x_1 + x_2 + 3x_3, \]
subject to
\[5x_1 + 2x_2 + 7x_3 = 420 \]
\[3x_1 + 2x_2 + 5x_3 \leq 280 \]

and
\[x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0. \]

1 (a) Using the two-phase method, work through phase 1 step by step.

C (b) Use a software package based on the simplex method to formulate and solve the phase 1 problem.

1 (c) Work through phase 2 step by step to solve the original problem.

C (d) Use a computer code based on the simplex method to solve the original problem.

4.6-9. Consider the following problem.

Minimize \[ Z = 3x_1 + 2x_2 + 4x_3, \]
subject to
\[2x_1 + x_2 + 3x_3 = 60 \]
\[3x_1 + 3x_2 + 5x_3 \geq 120 \]

and
\[x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0. \]

(a) Using the Big M method, work through the simplex method step by step to solve the problem.

1 (b) Using the two-phase method, work through the simplex method step by step to solve the problem.

(c) Compare the sequence of BF solutions obtained in parts (a) and (b). Which of these solutions are feasible only for the artificial problem obtained by introducing artificial variables and which are actually feasible for the real problem?

C (d) Use a software package based on the simplex method to solve the problem.

4.6-10. Follow the instructions of Prob. 4.6-9 for the following problem.

Minimize \[ Z = 3x_1 + 2x_2 + 7x_3, \]
subject to
\[-x_1 + x_2 = 10 \]
\[2x_1 - x_2 + x_3 \geq 10 \]

and
\[x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0. \]

4.6-11. Label each of the following statements as true or false, and then justify your answer.

(a) When a linear programming model has an equality constraint, an artificial variable is introduced into this constraint in order to start the simplex method with an obvious initial basic solution that is feasible for the original model.

(b) When an artificial problem is created by introducing artificial variables and using the Big M method, if all artificial variables in an optimal solution for the artificial problem are equal to zero, then the real problem has no feasible solutions.

(c) The two-phase method is commonly used in practice because it usually requires fewer iterations to reach an optimal solution than the Big M method does.