Optimization Theory (ISM 206) - Introduction

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1 Class topics

Main topics:
Linear Programming
Nonlinear programming
Heuristic Methods
Integer programming
Dynamic programming
Inventory Theory

Types of problems:
Linear: Linear functions for objective and constraints
Nonlinear: Nonlinear functions
Convex
Integer
Mixed-Integer
Combinatorial
Unconstrained: No constraints
Dynamic: Solved in stages

2 Optimization Overview

- Variables: \( x = (x_1, x_2, ..., x_n) \)
- Objective: \( \min f(x) \)
- Subject to Constraints: \( c_i(x) \leq 0 \); \( i \in E \)
- Sometimes additional constraints: Binary, Integer
- Sometimes uncertainty in parameters (stochastic optimization)

3 Types of Optimization Problems

Linear: Linear functions for objective and constraints
Nonlinear: Nonlinear Functions
Convex
Integer
Mixed-Integer
Combinational
Unconstrained
Dynamic: solved in stages

4 Definitions
Feasible region - area of possible outcomes/solutions
Solution (feasible point) vs. Optimal solution (best point) Global and local optimality -
Best point vs. a point better than others in immediate area
Optimality conditions - Duality - a related, complementary problem that can be solved
first to aid with the primary problem
Direct methods - methods using mathematical computations Numerical methods - methods
using analytical tools, guess and check Heuristics -

5 Stages of Optimization
1 Define problem and gather data
   - Feasibility check
2 Formulate mathematical model
3 Develop computer-based method for finding optimal solution
   - Design and Software implementation
4 Test and refine model
   - Validation
5 Prepare for ongoing model utilization
   - Training, installation
6 Implement
   - Maintenance, updates, reviews, documentation, dissemination