Image Formation,
Sampling, and Resolution

Perspective for the Day
Let's design a camera

- Idea 1: a piece of film in front of an object
- Do we get a reasonable image?

Pinhole camera

- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is known as the **aperture**
Pinhole camera model

- Pinhole model:
  - Captures **pencil of rays** – all rays through a single point
  - The point is called **Center of Projection** (focal point)
  - The image is formed on the **Image Plane**

Building a real camera
Camera Obscura

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

Source: A. Efros

Home-made pinhole camera

Why so blurry?

http://www.debevec.org/Pinhole/

Slide by A. Efros
Shrinking the aperture

- Why not make the aperture as small as possible?
  - Less light gets through
  - Diffraction effects…
Adding a lens

- A lens focuses light onto the film
  - Rays passing through the center are not deviated

A lens focuses light onto the film
- Rays passing through the center are not deviated
- All parallel rays converge to one point on a plane located at the focal length $f$
Adding a lens

- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
  - Other points project to a “circle of confusion” in the image

Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm
How can we control the depth of field?

- Changing the aperture size affects depth of field
  - A smaller aperture increases the range in which the object is approximately in focus
  - But small aperture reduces amount of light – need to increase exposure
Nice Depth of Field effect

Field of View / Focal Length

Sources: A. Efros, F. Durand
Sensor: An array of energy detecting devices

Example: CCD in a digital camera
Image Acquisition

**FIGURE 2.15** An example of the digital image acquisition process. (a) Energy (“illumination”) source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Image Sampling and Quantization

**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.
Image Sampling, Quantization and Noise

CCD vs. CMOS

- **CCD**: transports the charge across the chip and reads it at one corner of the array. An analog-to-digital converter (ADC) then turns each pixel's value into a digital value by measuring the amount of charge at each photosite and converting that measurement to binary form.

- **CMOS**: uses several transistors at each pixel to amplify and move the charge using more traditional wires. The CMOS signal is digital, so it needs no ADC.


Color sensing in camera: Color filter array

Bayer grid

Estimate missing components from neighboring values (demosaicing)

Why more green?

Human Luminance Sensitivity Function

Source: Steve Seitz

Demosaicing
Color sensing in camera: Prism

- Requires three chips and precise alignment
- More expensive

![Prism Diagram]

Color sensing in camera: Foveon X3

- CMOS sensor
- Takes advantage of the fact that red, blue and green light penetrate silicon to different depths

Source: M. Pollefeys

http://en.wikipedia.org/wiki/Foveon_X3_sensor

better image quality
Digital camera artifacts

- Noise
  - low light is where you most notice noise
  - light sensitivity (ISO) / noise tradeoff
  - stuck pixels

- In-camera processing
  - oversharpening can produce halos

- Compression
  - JPEG artifacts, blocking

- Blooming
  - charge overflowing into neighboring pixels

- Color artifacts
  - purple fringing from microlenses,
  - white balance

What Is An Image?

- Definition: An image is a 2-dimensional light intensity function, \( f(x,y) \), where \( x \) and \( y \) are spatial coordinates, and \( f \) at \( (x,y) \) is related to the brightness of the image at that point.

- Definition: A digital image is the representation of a continuous image \( f(x,y) \) by a 2-D array of discrete samples.

- The amplitude of each sample is quantized to be represented by a finite number of bits.

- Definition: Each element of the 2-d array of samples is called a pixel (Picture Element)
• An MxN digital grayscale image: (for color \( f(x,y) \) would be 1 color component)

\[
f(x,y) = \begin{bmatrix}
f(0,0) & \cdots & f(0,N-1) \\
\vdots & \ddots & \vdots \\
f(M-1,0) & \cdots & f(M-1,N-1)
\end{bmatrix}
\]

• Book Convention: (Same as MATLAB, except MATLAB starts index at 1)

\[
f(x,y) = \begin{bmatrix}
y \text{(columns)} \\
x \text{(rows)}
\end{bmatrix}
\]

**IMAGE SENSING AND ACQUISITION**

Two important components:

• *Illumination*

• *Sensing*

Illumination can be:

• *From the object* being imaged (e.g. Sun, radioactive decay, sound source, …)

• Independent source, but **reflected** by objects (e.g. most optical imaging, radar, sonar, ultrasound)

• Independent source, **transmitted** through objects (e.g. X-ray, geophysical borehole imaging)
Simple Model of (Gray) Image Formation
(Reflectance Model)

- Image \(0 < f(x,y) < \text{Inf}\), as a physical quantity related to measured energy

- Can distinguish two components:
  - **Illumination** incident on the object: \(0 < i(x,y) < \text{Inf}\)
  - **Reflectance** function of the object: \(0 < r(x,y) < 1\)
    - \(r = 0\) means total absorption, \(r = 1\) total reflection.

\[f(x,y) = i(x,y) r(x,y)\]

In other cases, \(r(x,y)\) may be replaced by the “transmissivity.”

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The Grayscale and Its Perception

- We call the intensity \(L = f(x,y)\) the “gray level” value

- The range of values of \(L_{\text{min}} < L < L_{\text{max}}\) is called the grayscale.

- Commonly, we use the range \([0, L-1]\) for integer \(L\) that is a power of 2.

Example: 3 bits, \(2^3 = 8\) gray levels
Number of Gray Levels and Contouring

Perception of Graylevel Contouring

Easy to see contouring in a ramp: 32, 64, 128, 256 (Typical)
Storage Needs for Images:

- Image MxN pixels, $2^B$ gray levels, $c$ color components
  - Size = $M \times N \times B \times c$
  - Example: MxN=1024x1280, B=8, c=3 (24 bit RGB image)
- Size = 31,457,280 bits (or 3.75 MBytes)

- Need to have (lossy) compression!

RESOLUTION

- Spatial Resolution:
  - Smallest discernable detail in an image.
  - Determined by spatial sampling.
  - Hard to measure objectively.
- Grayscale resolution:
  - Smallest discernable change in gray level
  - Even harder to measure.
Luminance change and its visibility

Weber’s Law: $\frac{\Delta I}{I} \approx 2\%$

Count the Black Dots (If you can)
Diffraction-Limited Imaging

\[ r = \frac{1.22 \lambda R}{D} = \frac{0.61 \lambda}{NA} = 1.22 \lambda F \]

\( r \) is the radius of the smallest resolvable feature in the image plane.

Alien-free Image Sampling

- To obtain an alias-free, diffraction limited image we need 4 pixels covering the Airy disk:

That is: radius of the Airy disk must match the pixel dimensions.
**Film, CCD’s, and the Eye**

**Image acquisition — photographic emulsion**
Photographic emulsions are generally made with photosensitive crystals of AgBr with grain size in the range 0.04-1.5 μm.

**Image acquisition — retina**
The human retina contains photoreceptors (rods and cones) whose dimensions and spacings are of the order of 10 μm.

**Film, CCD’s, and the Eye**

**Image acquisition — CCD camera**
CCD (charge coupled devices) chips are the basis of digital cameras and displays. CCD chips are fabricated with VLSI technology. The phototransistor is a solid-state, back-illuminated diode whose current is sensitive to light intensity.

Photomicrograph of a CCD camera surface showing portions of 16 unit cells each with dimensions 13 x 13 μm. Each unit cell corresponds to a pixel. The chip has 690 x 582 pixels in an area of 16 x 9.5 mm.

→ Looking at bright blue light:

\[
\frac{1.22\lambda R}{D} = \frac{1.22(0.5\mu m)(2cm)}{2mm} = 6\mu m
\]

**The image of a thin line is not a thin line**

<table>
<thead>
<tr>
<th>Image of a thin line</th>
<th>Pixel array</th>
<th>Film grains</th>
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**Notes by G+W**

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**Number of Pixels and Resolution**

**FIGURE 2.19** A $1024 \times 1024$ 8-bit image subsampled down to size $32 \times 32$ pixels. The number of allowed gray levels was kept at 256.

**Number of Pixels ≠ Resolution**

**FIGURE 2.20** (a) $1024 \times 1024$ 8-bit image. (b) $512 \times 512$ image resampled into $1024 \times 1024$ pixels by row and column duplication. (c) through (f) $256 \times 256$, $128 \times 128$, $64 \times 64$, and $32 \times 32$ images resampled into $1024 \times 1024$ pixels.
Number of Pixels $\neq$ Resolution

Read article “Myth of Megapixels” by David Pogue, available on course webpage.

FIGURE 2.25 Top row: images zoomed from $128 \times 128$, $64 \times 64$, and $32 \times 32$ pixels to $1024 \times 1024$ pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.