(1) Op amp and frequency response
Assume the op amp is ideal. (a) Derive an expression for the voltage gain of the circuit below as a function of \( w \). (b) Identify zeros and poles. (c) Sketch the magnitude Bode plot (label the axes).

![Circuit Diagram](image)

**Ideal Op Amp:**
\[
\begin{align*}
V_- &= V_+ (\text{connected to ground}) \\
I_- &= I_+ = 0
\end{align*}
\]

**KCL node \( V_c \):**
\[
I_1 = I_- + I_2 = 0 + I_2 = I_2 \quad \Rightarrow \quad \frac{V_m}{R} = \frac{-V_{out}}{Z_c} \quad \text{(2)}
\]

**Gain:**
\[
A_v = \frac{V_{out}}{V_{in}} = -\frac{Z_c}{R} = -\frac{V_{gaw}}{R} = -\frac{1}{jRCw} \quad \text{(1)}
\]

Poles & zeros: use Laplace variable \( s = jw \) \( \Rightarrow \)
\[
A_v = \frac{-1}{sRC} \quad \text{(3)}
\]

*Zeros: Numerator = 0 \Rightarrow \text{no zeros}*

*Poles: Denominator = 0 \Rightarrow \text{poles} \quad \text{at} \quad sRC = 0 \quad \text{at} \quad s = 0 \quad \text{(pole)}

There is a pole at \( s = 0 \)
\[
jw = 0, \quad \omega = 0
\]

In log scale
\[
\log 0 \rightarrow -\infty
\]

Pole is at \(-\infty\) in log axis

**Frequency Response:**
\[
\frac{20 \log |A_v|}{20 \log |A_v|} = 20 \text{dB/decade}
\]

\[
|A_v| \rightarrow w \quad \text{(log scale)}
\]
(2) CMOS inverter
Consider the CMOS inverter below with $V_{DD}=5V$. Assume that the MOSFETs have $K_{Pn}=50 \mu A/V^2$, $K_{Pp}=25 \mu A/V^2$, $V_{ton}=1V$, $V_{tsp}=-1V$, $(W/L)_n=3$, $(W/L)_p=6$, and $\lambda_n=\lambda_p=0$.

(a) If the input is high (5V), how much current does the output sink when $V_O=2V$? (i.e. find $I_D$)

(b) Suppose that $V_I=0$ and the output is accidentally connected to ground.
Determine the power supply current and the power dissipated in the inverter.

\[ V_{in} = 5V, \quad V_{out} = 2V \]

**NMOS**

\[ V_{GSn} = 5V > V_{to} \quad \text{threode} \quad \text{regime} \]

\[ V_{DS} = 2 < V_{GSn} - V_{to} \]

**PMOS**

\[ V_{GS} = 0 \Rightarrow \text{cut off} \quad \text{regime} \]

**NHOS**

\[ I_D = K \left\{ 2 \left( V_{GSn} - V_{to} \right) V_{DS} - V_{DS}^2 \right\} \]

\[ K = \frac{1}{2} (K_{Pn})_n \frac{W}{L} = 75 \frac{mA}{V^2} \]

\[ I_D = 75 \frac{mA}{V^2} \left\{ 2 (5-1) 2 - 2^2 \right\} = 75 \frac{mA}{V^2} (12) = 9 \text{ mA} \]

\[ V_{in} = 0, \quad V_{out} = 0 \]

**NMOS**

\[ V_{as} = 0 \Rightarrow \text{cut off} \]

**PMOS**

\[ V_{as} = -5 \Rightarrow V_{DS} = -5 \Rightarrow \text{saturation} \]

\[ I_D = \frac{1}{2} (K_{Pn})_n \frac{W}{L} = 25 \times 6 = 75 \frac{mA}{V^2} \]

\[ I_D = 75 \left( -5 - (-1) \right)^2 = 75 \times 16 \text{ mA} = 1.2 \text{ mA} \]

**Power dissipated in inverter**

\[ V_{DD} \times I_D = 5 \times 1.2 \text{ mA} \]

\[ = 6 \text{ mW} \]
(3) The figure below shows a logic inverter based on the differential pair. Here, Q1 and Q2 form the differential pair, whereas Q3 is an emitter follower that performs two functions: It shifts the level of the output voltage to make $V_{OH}$ and $V_{OL}$ centered on the reference voltage $V_R$, thus enabling one gate to drive another and it provides the inverter with a low output resistance. All transistors have $V_{BE} = 0.7V$ at $I_C = 1mA$ and have $\beta = 100$.

**Transistor:** \[I_C = I_s \exp(V_{BE}/V_T), V_T = 0.026V\]

**Differential Amplifier:** \[I_R = I_s/(1+\exp((V_{BE}-V_{B1})/V_T)), I_s = 1mA\]

a. For $V_I$ sufficiently low so that $Q_1$ is cut off, find the value of the output voltage $V_o$. This is $V_{OH}$.
b. For $V_I$ sufficiently high so that $Q_1$ is carrying all the current $I$, find the output voltage $V_o$. This is $V_{OL}$.
c. Determine the value of $V_I$ that results in $Q_1$ conducting 1% of $I$. This can be taken as $V_{IL}$.
d. Determine the value of $V_I$ that results in $Q_1$ conducting 99% of $I$. This can be taken as $V_{IH}$.
e. Sketch and clearly label the breakpoints of the inverter voltage transfer characteristics. Calculate the values of the noise margins $N_{MH}$ and $N_{ML}$. Note the judicious choice of the value of the reference voltage $V_R$.

![Diagram of logic inverter](image)

\[V_{CC} = 5V\]

\[R_1 = 1k\Omega\]

\[v_0 - v_I - v_R = V_R = 3.64V\]

\[I = 1mA\]

\[10mA\]

\[V_{BE}/V_T = 0.7/0.026\]

\[I_s = 1mA\]

\[\Rightarrow I_s = 2.03 \times 10^{-8}A\]

\[\sqrt{\frac{\ln 2}{\beta+1}}\]

**A**

Q1 cut off \[\Rightarrow \text{Current going through } R_1 \text{ is the same as the current into base of } Q3\]

\[I_{B3} = \frac{10mA}{\beta+1} \approx 0.01 \text{ mA}\]

\[V_{B3} = 5V - 1k\Omega (0.01 \text{ mA}) = 4.99V \quad (1)\]

\[V_{BE3} = ?\]

\[I_{C3} = I_s \exp(V_{BE3}/V_T)\]

\[\Rightarrow V_{BE3}/0.026 = 0.76V \quad (2)\]

\[V_0 = V_{B3} - V_{BE3} = 4.99 - 0.76 = 4.23V = V_{OH}\]

**B**

\[I_{C1} = 1mA \Rightarrow V_{B3} \approx 5V - R_1(I_{C1} + I_{B3}) = 3.94V\]

\[V_0 = V_{B3} - V_{BE3} = 3.94 - 0.76 = 3.23V = V_{OL}\]
(c) $Q_1$ conducting 1% of $I \Rightarrow I_{E_1} = \frac{1 \text{ mA}}{100} \times 1 \text{ mA} = 0.01 \text{ mA}$

$$I_{E_1} = \frac{1 \text{ mA}}{1 + e^{(V_{B2} - V_{B1})/V_T}} = \frac{1}{100} \times 1 \text{ mA} \Rightarrow 100 = 1 + e^{(V_{B2} - V_{B1})/V_T}$$

$$\Rightarrow V_{B2} - V_{B1} = V_T \ln(99) \Rightarrow V_{B1} = 3.52 \text{ V} = V_{IL}$$

(d) $Q_1$ conducting 99% of $I \Rightarrow I_{E_1} = 0.99 \text{ mA}$

$$\Rightarrow \text{similar way } V_{B1} = 3.76 \text{ V} = V_{IH}$$

$$NM_H = 4.23 - 3.76 = 0.47 \text{ V}$$

$$NM_L = 3.52 - 3.23 = 0.29$$

$$5 \text{ V}$$

$$\begin{cases} 4.23 \text{ V} = V_{OH} \\ 3.64 \text{ V} = V_R \\ 3.76 \text{ V} = V_{IH} \\ 3.52 \text{ V} = V_{IL} \\ 3.23 \text{ V} = V_{OL} \end{cases}$$
4. Cascode Configuration

The cascode configuration, also referred to as the common-emitter – common-base (CE-CB) connection, is shown in the figure below. Use the small-signal equivalent circuit (including the Early effect) to derive an expression for the input resistance, output resistance and the transconductance of the circuit. Remember that the output resistance can be calculated by shorting the input to ground and applying a test current at the output.

\[
\text{Transconductance } \frac{I_{out}}{V_{in}} \quad \text{(neglect Early effect to calculate)}
\]

Small signal model:

Input impedance

\[
R_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{\beta_i b_1} = R_{\Pi}
\]

Output impedance:

Circuit can be simplified to:

It is better to separate the two resistors, since we need to find \( \beta_i b_2 \).
KCL at nodes:
\[
\begin{align*}
KCL \ node \ V_i : \quad & \frac{0 - V_i}{r_{o1}} + \frac{0 - V_i}{r_{n2}} + \beta \dot{i}_{b_2} + \frac{V_x - V_i}{r_{o2}} = 0 \\
\dot{i}_{b_2} = \frac{0 - V_i}{r_{n2}} 
\end{align*}
\]  \tag{1}

\[
\Rightarrow \quad \frac{0 - V_i}{r_{o1}} - \frac{V_i}{r_{n2}} - \beta \frac{V_i}{r_{n2}} + \frac{V_x - V_i}{r_{o2}} = 0
\]

\[
V_i \left( \frac{1}{r_{o1}} + \frac{1}{r_{n2}} + \beta \frac{1}{r_{n2}} + \frac{1}{r_{o2}} \right) = \frac{V_x}{r_{o2}}
\]  \tag{eq. 1}

KCL at output
\[
\dot{i}_x = \beta \dot{i}_{b_2} + \frac{V_x - V_i}{r_{o2}}
\]

\[
\dot{i}_x = \beta \left( -\frac{V_i}{r_{n2}} \right) + \frac{V_x}{r_{o2}} - \frac{V_i}{r_{o2}} \quad \tag{1}
\]

\[
\dot{i}_x = -\left( \frac{\beta}{r_{n2}} + \frac{1}{r_{o2}} \right) V_i + \frac{V_x}{r_{o2}}
\]

Replace \( V_i \) from eq. 1:

If \( r_{o1} = r_{o2} \):
\[
\dot{i}_x = -\left( \frac{\beta}{r_{n2}} + \frac{1}{r_{o2}} \right)
\]

\[
\frac{1}{r_{o2}} - \frac{\left( \beta \frac{1}{r_{n2}} + \frac{1}{r_{o2}} \right) r_{o2}}{\left( \frac{2}{r_{o2}} + \frac{1+\beta}{r_{n2}} \right) r_{o2}} V_x + \frac{V_x}{r_{o2}}
\]  \tag{1}

Output impedance:
\[
\frac{V_x}{\dot{i}_x} = \frac{1}{\frac{1}{r_{o2}} - \frac{\left( \beta \frac{1}{r_{n2}} + \frac{1}{r_{o2}} \right) r_{o2}}{\left( \frac{2}{r_{o2}} + \frac{1+\beta}{r_{n2}} \right) r_{o2}}}
\]  \tag{1}

Transconductance:
\[
\frac{\dot{i}_{out}}{V_i} = A_{\text{transconductance}} =
\]

Let's neglect Early effect
\[
\dot{i}_{b_1} = \frac{V_i}{r_{n1}} \quad \tag{1}
\]

Node 1:
\[
\beta \dot{i}_{b_1} + (\beta \dot{i}_{b_2}) + (-\dot{i}_{b_2}) = 0 \quad \tag{2}
\]
\[
\dot{i}_{b_2} = \frac{V_x}{r_{n2}} \quad \tag{3}
\]

\[
\frac{1}{r_{n2}} \beta \left( \frac{V_i}{r_{n1}} \right) - \left( 1 + \beta \right) \left( \frac{V_x}{r_{n2}} \right) = 0
\]

\[
A_{\text{transcond}} = \frac{\dot{i}_{out}}{V_i} = \frac{\beta \dot{i}_{b_2}}{V_i} = \frac{\beta}{V_i} \left( -\frac{V_x}{r_{n2}} \right) = -\frac{\beta}{r_{n2}} \frac{V_x}{V_i} + \frac{\beta}{r_{n2}} \frac{V_x}{r_{n2}} \frac{1}{(1+\beta) V_i}
\]  \tag{1}

\[
= \frac{\beta}{V_i} \left( -\frac{1}{r_{n2}} \right) + \frac{1}{V_i} \left( \frac{\beta}{r_{n2}} \right)
\]