1. **Learning Objectives**

   After successfully completing this laboratory workshop, including the assigned reading, the lab problems, and any required reports, the students will be able to:

   1. Determine the semiconductor type from the polarity of the Hall voltage, knowing the orientation of all fields and currents in the experimental arrangement.
   2. Calculate the carrier concentration and mobility from the magnitude of the Hall voltage and the known experimental variables (magnetic field and sample resistance).
   3. Explain the response of the charge carriers in a material to a magnetic field.
   4. Predict the Hall voltage which would develop under a given set of Hall experiment variables.

2. **References**


3. **Theory**

   3.1. **Hall Effect**

   The Hall effect occurs because a charged particle moving in a magnetic field is subject to the Lorentz force given by:

   \[ F_{\text{Lorentz}} = q \mathbf{v} \times \mathbf{B} \]  (1).

   \[ |F_{\text{Lorentz}}| = q \mathbf{v} B \sin(\theta) \]  (1b) \((\theta)\text{ is the angle between }\mathbf{B}\text{-field and velocity vector }\mathbf{v}\).
Where \( q \) is a signed quantity representing carrier charge, \( \mathbf{v} \) is the particle velocity vector, and \( \mathbf{B} \) is the vector magnetic field. The basic Hall measurement is performed on a semiconductor bar with an electric field applied along its long axis, and magnetic field applied perpendicular to it.

3.2. Hall Voltage

Examine the sample shown in Figure 1. A voltage \( V \) is applied, giving rise to a field \( E_x = \frac{V}{L} \).

If the sample is n-type, the majority carrier electrons will move opposite the applied electric field, right to left (-\( x \)). The \( \mathbf{v} \times \mathbf{B} \) cross product is in the positive \( y \) direction for a \( \mathbf{B} \) field directed upward in the page. The carrier charge in this case is negative so the force is actually in the -\( y \) direction. This force causes the majority carrier electrons to be pushed towards the front edge of the sample. The entire sample remains neutrally charged as the positive charges of the donor ions are now uncompensated at the back. There is, however, a gradient of charge increasing from back to front giving rise to a second electric field perpendicular both to the externally applied electric field \( E_x \) and the \( \mathbf{B} \)-field. This new electric field opposes further accumulation of electrons (it could be viewed as rejection by electrons already there). The system reaches equilibrium when the force applied on carriers by the second electric field \( E_y \) equals and opposes the forced due to the \( \mathbf{B} \) field.

\[
q |E_y| = q |\mathbf{v} \times \mathbf{B}| \quad (2)
\]

\( E_y \) gives rise to a voltage which can be measured from the front to the back face of the sample. This is called the Hall voltage \( V_H = E_y w \) (see Fig. 1 for the definition of \( w \)). In this n-type sample, the voltage measured from front to back in the sample will be negative. If any of the sign definitions change, however, this sign may change too.

Now consider a p-type sample. Here the majority carrier holes move with the applied electric field, left to right in Figure 1. The force due to the magnetic field, \( q \mathbf{v} \times \mathbf{B} \) is in the -\( y \)- direction and once more carriers are crowded to the front face of the sample resulting in an electric field.
This time, however, because the carriers are positive, the Hall voltage measured from front to back on the sample will be positive. Thus, the majority carrier type determines the sign of the Hall Voltage.

![Diagram of Hall voltage circuit]

**Figure 1:** Hall voltage circuit

The velocity we have been discussing is the carrier drift velocity and is related to the current by:

\[ I_x = qnv_x A \quad \text{or} \quad I_x = qpv_x A \tag{3} \]

for n-type or p-type sample respectively. Remember that \( q \) is a signed quantity. In Equation 3, \( I_x \) is the current in the \( x \)-direction due to the applied electric field, \( n \) or \( p \) is the carrier concentration per cm\(^3\), and \( A \) is the cross sectional area of the sample in cm\(^2\) (width times thickness: \( w.t \)). The quantity \( v \) can easily be solved for and the result substituted in Equation 2, resulting in:

\[ E_y = -\frac{I_x B_z}{q |nA} \quad \text{(n-type)} \]

\[ E_y = +\frac{I_x B_z}{q |pA} \quad \text{(p-type)}. \tag{4} \]
in which the sign of the charge carrier is now expressed explicitly.

If we substitute the product \( wt \) for \( A \) in Equation 4, where \( w \) is the sample width and \( t \) the sample thickness, we can multiply Equation 4 by the width to get the Hall voltage:

\[
V_H = w E_y = \frac{-I_x B_z}{|q| n t} \text{ (n-type)}
\]

\[
V_H = w E_y = \frac{+I_x B_z}{|q| p t} \text{ (p-type)} \tag{5}
\]

Since \( V_H, B_z, I_x, t \) and \( q \) are all known (by measurement), it is possible to solve for the carrier concentration \( n \) or \( p \), and determine whether the sample is n-type or p-type.

3.3. **Hall Coefficient**

A useful measurement concept is Hall Coefficient, which is defined as:

\[
R_H = \frac{E_y}{J_x B_z} \tag{6}
\]

Writing \( R_H \) in terms of measurable quantities, we get:

\[
R_H = \frac{V_H t}{I_x B_z} \tag{7}
\]

If we substitute the value of \( V_H \) from Equation 5 into Equation 7, we can see:

\[
R_H = \frac{-1}{|q| n} \text{ (n-type)}
\]

\[
R_H = \frac{+1}{|q| p} \text{ (p-type)} \tag{8}
\]

The above analysis relies upon the idea that all carriers travel with the drift velocity. This is actually only an average velocity with the actual velocities being distributed over a Maxwellian distribution. Averaging in the presence of the magnetic field is more complicated than with electric field alone. One can compensate for this effect by writing the Hall coefficient as:
\[ R_H = \frac{-r}{|q| n} \quad \text{(n-type)} \]

\[ R_H = \frac{+r}{|q| p} \quad \text{(p-type)} \quad (9) \]

where \( r \) is a correction factor used to account for the difference between carrier velocity under influence of a magnetic field and the usual drift velocity. Typical values of \( r \) are found in Table 1:

<table>
<thead>
<tr>
<th>Value of ( r )</th>
<th>Dominant scattering mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.18</td>
<td>Lattice scattering</td>
</tr>
<tr>
<td>1.93</td>
<td>Impurity scattering</td>
</tr>
</tbody>
</table>

**Table 1.** Velocity correction factors for different scattering mechanisms

### 3.4. Hall Mobility

After we have determined the carrier concentration and type from Equations (7) and (9), we can use some further experimental information to determine the carrier mobility. We can measure the conductivity from Ohm’s law:

\[ \sigma = \frac{L}{A \times R} \quad (10) \]

\[ \sigma = |q| \mu_n n \quad \text{or} \quad \sigma = |q| \mu_p p \quad (11) \]

which means that the determination of carrier concentration along with the conductivity provide a measurement of the mobility. The mobility measured in this fashion is typically higher than the actual mobility, unless the factor \( r \) is applied to the computation of carrier concentration in Equation 9.
4. **Experimental Procedure**

The sample setup for the experiment is shown in Figure 2. We will be using a thin highly doped layer of silicon on a lightly doped silicon substrate of the opposite type. Thus only the highly doped region will be conducting the current. The sample configuration is shown from top view in (a) and in side view in (b). The test circuit is shown in Figure 3.

1. Measure and record the given Si dimensions for calculation the conductivity and Hall Coefficient.
2. Connect the circuit of Figure 3.
3. Make a sketch in your lab notebook showing the sample and its orientation in the magnetic field, so that you know the direction of the applied field \((V/L)\), the magnetic field \((B)\), and the Hall field \((E_y/w)\).
4. Measure the resistance of the sample by applying a voltage end-to-end and measuring the resulting current.
5. Measure the Hall voltage at three different magnetic field strengths (its easiest to do the strongest first) for three different sample currents (keep current below 5 mA). Measure the magnetic field using the Gaussmeter for each setting.
6. Note: To eliminate errors due to sample asymmetry, repeat the measurement with the sample reversed in the magnetic field. Averaging these two readings will cancel the asymmetries in contact placement.

7. Note: 1 kilogauss = $10^{-5}$ Weber/cm$^2$. Use Weber/cm$^2$ in all equations to obtain concentration in cm$^{-3}$.

**Figure 3.** Circuit setup for Hall Effect test.
5. **Written Report**

Your written report should include the following sections:

1. Title Page.

2. Introduction – Explain the Hall effect (theory, include relevant equations, define all quantities).

3. Procedure – Explain what you did, rather than copying from the lab notes. Include a sketch.

4. Data Analysis and Results - Show the equations which you used to calculate the carrier concentration, mobility and type. Carefully state the results of your measurements and calculations. Your results include the sample resistance, the Hall mobility, Hall coefficient, semiconductor type, and carrier concentration.

5. Discussion of results – Determine whether the results of your data are appropriate, i.e. whether the calculated values make sense. Discuss the results with your group and whether they agree with theoretical values. You’ll have to do some reading in the textbook to determine whether your results are reasonable or not.
PROBLEMS

Date __________
Group Members ______________________________________

BE SURE TO REPRODUCE THIS DISCUSSION IN YOUR LAB NOTEBOOK FOR FUTURE REFERENCE!!

1. Sketch the set-up of the circuit and the sample, by inspection (not by copying out of the lab notes). Include a right-handed coordinate system on the sketch, and indicate all directions of current and voltage in terms of this coordinate system. You will have to look at the sample itself to be sure you have it oriented correctly in your sketch.

Answer the following questions, in terms of your diagram above.

2. In which direction does the applied current flow? (Give your answer as a vector i.e \( \mathbf{I}_x \) or \(- \mathbf{I}_y\).
(Note: With low on Lead 1 and high on Lead 2, a positive voltage means potential is higher at 2 then at 1; E-field points from 1 to 2; electron moves from 2 to 1; thus conventional current is from 1 to 2 in direction of positive voltage.)

3. What is the voltage across the length of the sample? What is the resistance \( R \) of the sample? What is the resistivity \( \rho \) of the sample? What is the conductivity \( \delta \) of the sample?

4. In which direction does the magnetic field point?
(Note: B-field lines point from N to S.)

5. In which direction is the Hall voltage?

6. What is the Hall coefficient? What is the carrier type? What is the carrier concentration?

7. What is the mobility of the carriers?
Magnetic Induction vs Coil Current

TA: Need to recalibrate this curve for our electromagnet (need a Gaussmeter)
Grading Guidelines for Laboratory Report

Experiment 3: Conductivity, Mobility, and Carrier Concentration measured by Hall Effect

Student Name _________________________

Score

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<thead>
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<th>Writing Style &amp; Structure</th>
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<td>Spelling &amp; Neatness</td>
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<tr>
<td>Introductory (Background) Section [Overall]</td>
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<tr>
<td>• Hall mobility, conductivity, carrier concentration</td>
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<td>• Hall effect-geometry, Lorentz force</td>
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<td>Experimental (Procedure) Section</td>
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<td>• Includes sketch</td>
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<td>Results</td>
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<td>• Figures Presentations (use appropriate Graphics, Labels,)</td>
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<td>• Distinguishes data from results and reports results (n or p, (\mu_H, \sigma), etc.)</td>
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<td>• Measured Data Presented in tabular or graphical format</td>
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<td>Discussion</td>
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<td>• Makes some attempt to determine whether results are correct or sensible, links results to theory presented earlier</td>
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<td>Conclusion</td>
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**Total Report Score = Sum of above/100**