1.0 Learning Objectives:

After successfully completing this laboratory workshop, including the assigned reading, the lab problems, and any required reports, the students will be able to:

1. Determine the thermal coefficient of resistivity for a metal from a linear regression of measured data (using e.g. Excel).
2. Explain why a metal’s conductivity changes with temperature.
3. Distinguish between impurity and lattice scattering, as well as between the effects of temperature, on the different types of scattering.
4. Explain the relationships between conductivity, resistivity, mobility, scattering time, mean free path and carrier concentration.
5. Distinguish between resistance and resistivity and be able to convert between them using the physical dimensions of a sample.

2.0 Reference:


3.0 Theory:

The conductivity of any material can be most simply described as a function of the density of charge carriers and the mobility of the various charge carriers. Metal are highly conductive because they have very high densities of free electrons. Semiconductors are less conductive because they have fewer charge carriers. Much information can be gained about a material by
studying the effect of temperature on the conductivity. Temperature may effect the density of charge carriers, as in a semiconductor, or it may only affect the mobility of the carriers. The behavior of metals and semiconductors can be differentiated by their conductivity response to temperature. The experiments to be performed this week and next week should provide the student with insight into the different characteristics of metals and semiconductors.

3.1 Conductivity and Resistivity:

The conductivity of a material is defined by Ohm’s law [text p. 104]:

\[ J = \sigma \mathbf{E} \]  \hspace{1cm} (1)

Where \( J \) is the current density (Amps/cm\(^2\)) and \( \mathbf{E} \) is the electric field in the direction of current flow (Volts/cm). This is a different form of Ohm’s Law than you are probably familiar with.

Current density and electric field cannot be measured directly with our lab equipment. Therefore, it is useful to express conductivity in terms of familiar voltage and current. The current density is given by [text p.100]:

\[ J = \frac{I}{A} \]  \hspace{1cm} (2)

where \( I \) is the current in Amps and \( A \) is the cross sectional area in cm\(^2\). The electric field is given by:

\[ E = \frac{V}{L} \]  \hspace{1cm} (3)

where \( V \) is the total voltage drop along the sample and \( L \) is the length of the sample in centimeters. Equation 4 is valid as long as the sample is uniform, and there is no voltage drop in the contacts.

Rearranging Equation 1 and substituting Equations 2 and 3 gives:
\[ \sigma = \frac{1}{\rho} = \frac{J}{E} = \frac{IL}{AV} = \frac{1}{R} \cdot \frac{L}{A} \quad (4) \]

where \( \rho \) is the sample resistivity (in \( \Omega \) cm) and \( R \) is the sample’s measurable resistance (in \( \Omega \)).

Thus by measuring the resistance directly (or the voltage and current), and knowing the sample’s physical dimensions, we can calculate the sample’s resistivity or its conductivity. Typically, resistivity is reported for metals, and conductivity is reported for semiconductors. Conductivity and resistivity are reciprocally related. The length of a sample is the dimension along which the current flows; the cross sectional area is taken on the plane normal to the current flow.

3.2 Conductivity of a Metal

For a metal, the electrical conductivity is given by [text p.104: \( \sigma = en\mu_d \quad (5) \)]

where \( \mu_d \) is the electron mobility, \( e \) is the charge on the electron, and \( n \) is the density of free electrons. The latter number is determined by the number of valence electrons per atom \( (Z) \) multiplied by the atomic density of the metal [calculation shown in example 2.2 in the text.]. The free electron density does not change with temperature for a metal: only the mobility \( \mu \) varies with temperature. At temperature more than a few Kelvins above absolute zero, the mobility of electrons in a metal decreases linearly with temperature. Thus the conductivity decreases linearly with temperature.

For metals, we usually discuss the resistivity instead of the conductivity. These two properties are reciprocally related, and \( \rho = \frac{1}{\sigma} \). The value of \( \rho \) depends on the structure of the metal and the temperature. The value of resistivity increases linearly with temperature, due to the change in the value of mobility.
3.3 Effects of Temperature on a Metal

Matthiesen’s rule suggests that a metal’s resistivity can be viewed as comprised of two components, one due to impurities and defects, and the other due to lattice vibrations, as [text p.111]:

\[ \rho = \rho_{\text{res}} + \rho_{\text{th}} \]  

(6)

The magnitude of \( \rho_{\text{res}} \) called the residual resistivity is determined by the defects within the structure of the material: the concentration and distribution of vacancies, dislocations, impurities and grain boundaries. Thus, a single crystal of copper would have a lower residual resistivity than polycrystalline copper, or copper alloyed with nickel. These defects cause an increase in the scattering of electrons, thus reducing the mean free time between collisions.

The magnitude of the second term, \( \rho_{\text{th}} \), depends on the temperature of the metal, and is due to the presence of lattice vibrations. The amplitude of the lattice vibrations increases with increasing temperature, and the more they interfere with conduction.

An empirical equation describing this effect is:

\[ \rho = \rho_0 \left[ 1 + \alpha_0 (T - T_0) \right] \]  

(7)

where \( \rho_0 \) is the resistivity in (in \( \Omega \cdot \text{cm} \)), and \( \alpha_0 \) is the temperature coefficient of resistivity, (in \( ^\circ\text{C}^{-1} \)), defined at a reference temperature. A plot of the resistivity of a metal versus temperature, according to Equation 7, should be linear. The values of \( \rho_0 \) and \( \alpha_0 \) for different metals can be determined from the experimental data. The reference temperature is usually either \( 0^\circ\text{C} (273\text{K}) \) or \( 20^\circ\text{C} (293\text{K}) \) [see text Figs. 2.6, 2.7 p.114-115].

4.0 Experimental Procedure

In this experiment you will measure the resistance of a copper wire as function of temperature.

The sample is a coil of insulated copper wire, wound around a glass tube, coated with epoxy, and
immersed in a flask of water. You’ll heat the water from 0°C to 100°C and record the resistance using a digital multimeter in resistance mode.

Using the physical dimensions of the wire, you can calculate the resistivity of copper in the temperature range. Your experimental objective is to fit a linear equation to your data, and calculate values for $\rho_0$ and $\alpha_0$ for copper. Look at Equation (7) carefully to be sure you have correctly fitted your data. Then compare your measured results to values found in the textbook or other reference books.

5.0 Report

Your written report should include the following sections:

1. Title Page.
2. Introduction – Explain how the metal’s resistivity depends on temperature, and why.
3. Procedure – Explain what you did, rather than copying from the lab notes. Include a sketch.
4. Data Analysis and Results – Show the equations that you used to calculate the resistivity. $\rho_0$ and $\alpha_0$. Carefully state the results of your measurements and calculations.
5. Discussion of results – Determine whether the results of your data are appropriate, i.e whether the calculated values make sense. Discuss the results with your group and whether they agree with theoretical values. You’ll have to do some reading in the textbook to determine whether your results are reasonable or not.
Grading Guidelines for Laboratory Report
Experiment 2
Temperature Coefficient of Resistivity for Metals

Student Name _________________________

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<td>Results</td>
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<td>Figure Presentations (use appropriate Graphics, Labels)</td>
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<td>Distinguishes data from results and reports results ($\alpha$, $\rho_o$)</td>
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<td>Discussion</td>
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<td>Make some attempt to determine whether results are correct or sensible, links results to theory presented earlier, and values to textbook information</td>
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<td>Conclusion</td>
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Lab 2: Temperature Dependence of Metal Resistivity
Laboratory 2 Problems

Date __________ Section Day ___________
Group Members ____________________________________________

This sample consist of a coil of “magnet wire”, which is 30-gauge copper wire, coiled around a test tube. Coated with epoxy and immersed in a water flask. The flask is heated on a hot plate. Thus the temperature of the water and the wire can be changed from room temperature to 100°C. A thermometer is also immersed in the water bath to monitor the temperature. Power supply, voltmeter and current meters are also attached. Use as low a current as possible at which you can still measure a steady voltage. An appropriate value is about 5mA. The hot plate should be set so it doesn’t heat too fast. As the wire slowly warms up, the voltage can be recorded at each temperature. Make a data table.

This information can then be used to calculate the resistivity, then plotted versus ∆T to find the temperature coefficient of resistivity ($\alpha$).

The sample dimensions are as follows: 25 feet of 30-gauge wire (based on the diameter of the tube and number of turns you can calculate the approximate length of the copper wire). The diameter of the wire (needed to find the resistivity) can be found from the wire gauge chart. The chart also lists the resistance per unit length, and students can use this information to compare to their own measurements.

After you have taken your data, answer the following questions:

- How will you convert your raw data (either V and I, or R) into resistivity?
- What should you use for the axes of your scatter plot?
- What reference temperature will you use?
- After you perform a linear regression on your data, what is the meaning of the slope and intercept?

This problem requires you to correctly fit your data using linear regression. Have your lab instructor check over your results to be sure you have done the work correctly. You will receive full marks for correct results and procedure; and zero marks for incorrect. So stay around until you get it right!