Maxwell's eqns:

\[ \nabla \cdot \mathbf{D} = \rho_v \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]

\( \mathbf{E} \) = elec field intensity (volts/m) - due to distrb. of elec charge
\( \mathbf{D} \) = elec flux density (wbd/m^2) = \( \varepsilon \mathbf{E} \) - due to polarization of material
\( \mathbf{B} \) = mag flux density (Tesla) - due to elec. currents
\( \mathbf{H} \) = mag field intensity (amp/m) = \( \frac{\mathbf{B}}{\mu} \) - due to magnetization

\( \varepsilon \) = elec permittivity (Farads/m)
\( \mu \) = mag permeability (H/m)

The key here is that the elec and mag fields are intertwined
ie, \( \mathbf{E} \) \& \( \mathbf{B} \) related, \( \mathbf{H} \) \& \( \mathbf{D} \) related as well

As long as, \( \mathbf{B} \) and \( \mathbf{D} \) vary with time!

What we will use in this case when \( \mathbf{B} \neq \mathbf{D} \)
don't vary with time!!

\[ \frac{\partial \mathbf{B}}{\partial t} = 0, \quad \frac{\partial \mathbf{D}}{\partial t} = 0 \]

Then we can study, elec \& mag fields INDEPENDENTLY.
we start with the field of "electrostatics"

\[
\begin{align*}
\nabla \cdot \vec{D} &= \rho_v \\
\nabla \times \vec{E} &= 0
\end{align*}
\]

\[\vec{D} = \varepsilon \vec{E}\]

Examples of electrostatic applications:
- liquid crystal displays
- electrocardiogram
- inkjet printers
- xerox machines
- X-ray machines
- oscilloscopes

before we begin our study of electrostatics a few notes:

charge on individual electron, e-
\[1.6 \times 10^{-19} \text{ C} \]

on atomic scale, charge is DISCRETE, i.e., e- or p for EJM, generally we are talking about sizes smaller than atomic scales.

so here we can ignore the discreteness of charge.

i.e., we speak of amount (the flow of charge) as continuous flowing charge, and we speak of a charge within an elemental volume as if it were uniformly distributed within that volume.

i.e., charge density \[\rho_v = \lim_{\Delta v \to 0} \frac{\Delta q}{\Delta v} = \frac{dq}{dv} / \text{C/m}^2\]
Let's do a few examples of unit charge distribution calculus.

1. Ring of charge
   calc. $E$ at $P(0,0,z)$

\[ \nabla \times \vec{E} = \frac{\rho S \, dS}{4\pi \varepsilon_0} \]

\[ \vec{E}_r = \frac{\rho S r \, dr \, dl}{4\pi \varepsilon_0 (r^2 + z^2)^{3/2}} \]

\[ \vec{E}_z = \frac{\rho S r \, dr \, dl}{4\pi \varepsilon_0 (r^2 + z^2)^{3/2}} \]

There is no net $E_\phi$ because of the symmetry.
21st Oct with Let $\theta = 135$

now we need to integrate up all the dq around the ring.
but $1^{st}$ - use symmetry to simplify things. II always had for symmetry.

**NOTE** - on opposite side of ring

diff $E$ is identical except for a $\pi$ wrap in opp. direction.

so these ramps will cancel.

$\vec{dE}_1 = -\vec{dE}_2$

so net for the $2$ diametrically

opposed elements is $\vec{dE}_1$ and $\vec{dE}_2$ are in same direction. $\pi$.

$\vec{dE}_{12} = \int_0^\pi \left( \frac{1}{4 \pi \varepsilon_0} \frac{(r \, dr \, d\theta) \, \vec{2}}{(r^2 + z^2)^{3/2}} \right) \sin \theta \, d\theta$

now we integrate, but note that we need only

$q_0 \to 0 \to \pi$ since we have the $2$ diametrically opposed parts here.

$\int_0^\pi \vec{dE} = \frac{1}{2} \int_0^\pi \left( \frac{1}{2 \pi \varepsilon_0} \frac{r \, dr \, d\theta \, \vec{2}}{(r^2 + z^2)^{3/2}} \right)$

$= \frac{1}{2} \int_0^\pi \left( \frac{1}{2 \pi \varepsilon_0} \frac{z (r \, dr \, d\theta)}{(r^2 + z^2)^{3/2}} \right)$

But total charge on ring is $Q = \int_0^\pi \int_0^{2 \pi} (r, \phi, z)$.

field due to ring is

$$\vec{E} = \frac{\lambda_0}{\varepsilon_0} \int_0^\pi \int_0^{2 \pi} \frac{r^2}{(r^2 + z^2)^{3/2}} \, dr \, d\theta \, Q$$

note extreme cases: 1) $z=0$, center of ring, $\vec{E} = 0$ (obvious from symmetry)

2) $z \gg R$, $\vec{E} \approx \frac{1}{2 \pi \varepsilon_0} \frac{Q}{4 \pi z^2}$

field at $z$ by $\frac{1}{2 \pi \varepsilon_0} \frac{Q}{4 \pi z^2}$ at $z$.
extend this result to a uniformly charged disk of radius \( R \) with total charge \( \pm \frac{q}{2} \). We have a ring from \( 0 \) to \( R \)

\[
\vec{E} = \frac{1}{2\varepsilon_0} \int_0^R \frac{\rho_s r \, dr}{(R^2 + z^2)^{3/2}} = \frac{1}{2\varepsilon_0} \left[ \frac{R^2}{(R^2 + z^2)^{1/2}} \right]_0^R
\]

so here we have an example where you must make math choices based upon the physical reality of the situation.

Note: as \( R \to \infty \) (an infinite sheet of charge),

\[
\vec{E} \to \pm \frac{\rho_s}{2\varepsilon_0}
\]

we call this a uniform field (it doesn't depend upon distance away)

\[
\uparrow E \uparrow \uparrow \uparrow \quad \rho_s
\]

\[
\downarrow \downarrow \downarrow \downarrow \quad \text{everywhere}
\]

and if we go far away from the disk, \( \| z \| \to \infty \)

\[
\vec{E} = \frac{1}{2\varepsilon_0} \frac{q}{2\pi R^2} \left[ 1 - \frac{1}{R^2 + z^2} \right] = \frac{1}{2\varepsilon_0} \frac{q}{2\pi R^2} \left[ 1 - \frac{1}{\sqrt{1 + z^2 / R^2}} \right]
\]

\[
\to \frac{q}{4\pi \varepsilon_0} \frac{1}{z^2} \quad \text{like the charge}
\]
finally lets do uniformly charged 1 rad sphere

with total charge $Q = \frac{4}{3}\pi r^3 \rho_v$

one can see that we could go thru same steps

taking diff we element of charge $dQ = \rho dV$

with $dV = r^2 \sin \theta \, d\theta \, d\phi \, dr$

and then calc the
diff field from each element

integrating over the volume.

- would get fairly complex.

sometimes

to find E

there is an easier way to fields

Gauss's law

so take volume integral of both sides

the div of D is the vol charge


de = \epsilon \vec{E}

so take volume integral of both sides

$\int \nabla \cdot \vec{D} \, dV = \int \rho_v \, dV = Q$  the charge enclosed by the volume.

remember last week

but div theorem of vector calc says for any vector field, $\vec{A}$

$\int_v \nabla \cdot \vec{A} \, dV = \int_{\partial V} \vec{A} \cdot dS$

vol integ of div = total outward flux thru surf each S

$\int_v \nabla \cdot \vec{D} \, dV = \int_{\partial V} \vec{D} \cdot dS = Q$

embod charge

Gauss' law

note that is for any surface!  ANY

Surface -
doesnt have to

be a physical

surfaced
so lets use Gauss' law to calc $E^2$ due to

uniformly charged sphere pt change to see how it un

take unit sphere of radd R Rad a Q (not reading)

we know $D = \varepsilon E$ radially

out ward from pt charge

$\vec{D} \cdot d\vec{s} = D ds$

since $\theta(a, \vec{D}, ds) = 0$

\[ \int_S \vec{D} \cdot d\vec{s} = \int_R \vec{D}_r \cdot \hat{r} ds = \int_0^R D_r ds = D_r 4\pi R^2 = Q \]

only rad unp

$D_r = \frac{Q}{4\pi R^2} = \varepsilon E$

\[ E = \frac{\varepsilon}{4\pi R^2} \]

note if it is only unp in directionl

surf that counts in this

surf integral.

we choose inf so that sym consideration allow

(31) only , t at inf.

lets now apply to uniform charged sphere (we put did)

$P_v = \frac{Q}{4\pi R^3}$

total charge what a field at r out ind

sphere.

take "Gauss inf" as sph rad $R > R$

only radul $E$

\[ \int_S \vec{D} \cdot d\vec{s} = \int_0^R E E \hat{r} ds = EE 4\pi R^2 = Q ! \]

I leave it to you to find

field inside this sphere

\[ E = \frac{Q}{4\pi \varepsilon R^2} \]

indeed take pt charge to surf and out side sphere \( Q \) unifadd
Let's look at the charge within a conductor, using Gauss' law

\[ \text{Remember field due to \textbf{no} charged sheet} \]

\[ \begin{align*}
\text{surf charge density} &= \sigma \quad \uparrow \uparrow \uparrow \\
\text{uniform} &= \underline{\vec{E} \text{ uniform} + \text{surf}} \\
\uparrow \uparrow \downarrow \downarrow \downarrow
\end{align*} \]

\[ \vec{E} = \frac{\sigma}{2\varepsilon} \]  
\[ \text{pts up on top} \]
\[ \text{pts down below} \]

Consider 2 planes of charge - same surf charge density

\[ \uparrow \quad \text{same } \vec{E} \quad \text{everywhere} \]
\[ \downarrow \]

But stuff in between cancels!

\[ \uparrow \quad \text{therefore } \vec{E} = 0 \text{ inside!} \]
\[ \downarrow \]

Another way of \( \vec{E} = 0 \) inside: change on inf would move - no longer a "static" case -

So given that \( \vec{E} = 0 \) inside a conductor, what does Gauss' law tell us about charge?

\[ \begin{align*}
\text{surf} \quad \vec{E} &= 0 \quad \text{everywhere} \\
\int_{\text{surf}} \vec{E} \cdot d\vec{s} &= \oint_{\text{surf}} \sigma \cdot d\vec{A} = Q_{\text{enclosed}} \\
\text{and surf} \quad \text{unind surf} \\
\rightarrow \quad \text{all even charge resides on} \quad \text{surf of conductor!}
\end{align*} \]

\[ \text{note} \]

\[ \text{I put the inf.!} \]
\[ \text{unless it is outside.} \]
we know law to find field due to long line of uniform charge

\[ \text{line charge density} = \rho \]

\[ \rho = \frac{dq}{dl} \]

we want field at \( r \) from wire

- take a cyl surf of rad \( r \)
  and length \( l \).

because of sym, we know field \( \vec{D} = \varepsilon \vec{E} \) \( \perp \) wire

- only an \( \hat{r} \) comp

\[ \vec{D} = \hat{r} \cdot \frac{dq}{dl} \]

total enclosed charge \( Q = \rho \cdot l \)

since only \( \hat{r} \) comp to \( \vec{D} \), then \( \perp \) surf

\[ \oint \vec{D} \cdot d\vec{s} = \int \hat{r} \cdot r^2 (r \hat{\sigma} \hat{d}z) + \int \text{other comps cut top/bot} \]

\[ = \int \hat{r} \cdot r \hat{\sigma} \hat{d}z \]

\[ = \int_0^{2\pi} \int_0^l \hat{r} \rho \hat{\sigma} \hat{d}z \]

\[ = 2\pi \rho \int_0^l \hat{\sigma} \hat{d}z \]

\[ = 2\pi \rho \cdot l \]

\[ \hat{r} \cdot \hat{D} = 1 \]

\[ \hat{r} \cdot \hat{E} = 0, \quad \hat{\sigma} = 0, \quad \hat{\sigma} = 0 \]

\[ \vec{E} = \frac{\rho}{\varepsilon} \hat{\sigma} \]

\[ \vec{E} = \frac{\rho}{2\pi \varepsilon \hat{r}} \]

\[ \hat{r} = \text{line of charge} \]

leave it to student to unvime

you self see that you could have done this

by integrating \( \sigma \) over diff element

\[ 0, 0 \rightarrow \pi \]

\[ \text{wait pg} \]
Let's do it


\[ dE_r = \frac{dq}{4\pi \varepsilon_0 r^2} \]

\[ \theta \]

\[ r \]

\[ z \]

\[ \theta \]

\[ \frac{\partial \theta}{\partial z} \]

\[ \frac{\partial \theta}{\partial z} \]

\[ dE \]

\[ dE = \frac{dq}{4\pi \varepsilon_0 r^2} \]

\[ dE_r = \frac{dq}{4\pi \varepsilon_0 r^2} \]

\[ \theta \]

\[ \frac{\partial \theta}{\partial z} \]

\[ \theta \]

\[ z \]

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\[ \frac{\partial \theta}{\partial z} \]

\[ dE_r = \frac{P}{4\pi \varepsilon_0 (r^2 + z^2)} \]

\[ dE_r = \frac{P}{4\pi \varepsilon_0 (r^2 + z^2)} \]

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NOTE II: 

\[ \mathbf{E} = -\nabla V \]

say \( \mathbf{E} \) in one direction then

\[ \text{grad} V = \frac{dV}{dq} = -\mathbf{E} \cdot \mathbf{dl} \quad \text{from before II} \]

\[ \therefore \int_{P_1}^{P_2} dV = -\int_{P_1}^{P_2} \mathbf{E} \cdot \mathbf{dl} = V_2 - V_1 = V_{21} \]

along any path, only ends count due to law of energy.

(e.g., gravity - walk up hill - PE expended)

same whether you walk straight up or on path that has "hill" element. since PE \( \propto \Delta h \) but diff only.

since pot indep of path then

\[ \oint \mathbf{E} \cdot d\mathbf{l} = 0 \quad \text{use} \quad V_{1 \rightarrow 2} = -V_{2 \rightarrow 1} \quad \text{another way of saying} \]

Kurzheff's law

we can define potential with respect to any pt in space -

generally defined with respect to some arbitrary

\[ 1eV_p = -\int_{\text{so}}^{P} \mathbf{E} \cdot d\mathbf{l} \]

energy to bring unit + charge from so to pt \( P \)
Example: potential due to pt charge.

Remember \( \mathbf{E} = \frac{\mathbf{q}}{4\pi \varepsilon_0 R^2} \) and law

\[
V = -\int_{P}^{E} \mathbf{E} \cdot d\mathbf{l} = -\int_{0}^{\infty} \frac{\hat{r} \cdot \mathbf{q}}{4\pi \varepsilon_0 R^2} \cdot \hat{r} dR = -q \int_{0}^{\infty} \frac{dR}{4\pi \varepsilon_0 R^2}
\]

\[V(\mathbf{r}) = \frac{q}{4\pi \varepsilon_0 R} \]

or of charge at some location other than origin say \( \mathbf{r}_1 \),

\[V(\mathbf{r}) = \frac{q}{4\pi \varepsilon_0 |\mathbf{r} - \mathbf{r}_1|} \]

and for ensemble of pt charges

\[V(\mathbf{r}) = \frac{1}{4\pi \varepsilon_0} \sum \frac{q_i}{|\mathbf{r} - \mathbf{r}_i|} \]

Note: we only deal with scalar quantities.

Similarly for unit distribution

\[V(\mathbf{r}) = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho_v d\mathbf{v}}{R} \]

\[= \frac{1}{4\pi \varepsilon_0} \int_{S} \frac{\rho_s ds}{R} \]

\[= \frac{1}{4\pi \varepsilon_0} \int_{\mathbf{r}} \frac{\rho_l dl}{R} \]

Note: all integrals are over appropriate domains.
another example of using potential to get E field.

```
1. elec dipole // 2 pt charges, equal mag/opp sign.

Consider ±q separated by d.

calc potential V at
dist R >> d //

\[ V_p = V_1 + V_2 = \frac{Q}{4\pi \epsilon R_1} + \frac{-Q}{4\pi \epsilon R_2} = \frac{Q}{4\pi \epsilon} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{Q}{4\pi \epsilon} \left( \frac{R_2 - R_1}{R_1 R_2} \right) \]

with notation that R >> d then R_2 - R_1 \approx d \times \epsilon \sim 0 \sim \epsilon

\[ V_p = \frac{Q}{4\pi \epsilon} \left( \frac{d \omega \epsilon}{R^2} \right) \]

note that \(d \omega \epsilon = \theta \cdot \hat{p} \cdot \hat{r} \) where \(\hat{p}\) is unit vector from

\[ \omega \epsilon \sim 0 \sim \epsilon \] when \(\hat{p} = q \hat{d} \) is called


\[ V = \frac{\hat{p} \cdot \hat{r}}{4\pi \epsilon \epsilon_0 R^2} \] from an electrostatic dipole


\[ \vec{E} = -\vec{\nabla} V = -\left( \frac{\hat{p} \cdot \hat{r}}{\epsilon_0 R^2} + \vec{e} \cdot \frac{1}{\epsilon_0 R} \frac{\hat{r}}{R} + \vec{e} \sin \theta \frac{\hat{\theta}}{\epsilon_0 R} \right) \]

we get

\[ -\frac{1}{\epsilon_0 R} \frac{\hat{p} \cdot \hat{r}}{R^2 \epsilon_0} + \frac{\hat{p} \cdot \hat{r}}{\epsilon_0 R^2} + \vec{e} \sin \theta = 0 \]

\[ \vec{E} = \frac{\hat{p} \cdot \hat{r}}{4\pi \epsilon_0 R^3} \left[ \hat{r} 2 \omega \epsilon + \hat{\omega} \sin \theta \right] \]

\(R >> d\) near field

\(R \sim d\) far field

\(R \ll d\) not useful

```