mention will diagnostic
follow recommendations
we will give only very brief reviews here

next week we will be starting our discussion of electric, magnetic fields -
there are free fields and quantities that have both magnitude and duration
they are vectors

so for we really only discussed scales quantities -
like current, voltage, impedance -
a scale can be expressed by its magnitude and phase angle.

(of it is a pos. real quantity, by its magnitude alone)

\[ z = x + jy \rightarrow |z| = \sqrt{x^2 + y^2}, \tan \theta = \frac{y}{x} \]

so \[ z = 12 \angle 0 \text{°} \]

because vectors have dimensions associated with them we need
to be sure you understand the math tools for manipulating them
in 3D space. some the results of the drag with text are in -
it will be useful to review vector algebra and vector calculus,
so that we won't be dropped down by the math later -

so to specify a vector, we need to specify its ends in 3D space
(along each of its 3 dimensions) there are several types
of and systems useful in studying vector quantities -
most common are rectangular, cylindrical, spherical -
we will discuss later, and you had a chance in lab #1
to do this as well.

remember, you can use any word system you want -
it is just some problems were easily relate in certain word systems
that reflect the sym. of the problem.
The review vector algebra. -

we write \( \vec{A} \) to be definition of a vector - in text it is just \textbf{BOLD}

\[ \vec{A} = \hat{A} |\vec{A}| \quad \text{where} \quad \hat{A} = \frac{\vec{A}}{|\vec{A}|} \quad \text{is the unit vector in the direction of} \quad \vec{A} \]

and \( |\vec{A}| \) is the absolute magnitude of \( \vec{A} \)

in short words (sometime called cartesian words)

the unit vectors (of the 3 dimensions) are

\[ \hat{x}, \hat{y}, \hat{z} \quad - \text{the directions of the three und. axes} \]

\( \vec{A} \) is represented in terms of the unds. along three orthogonal directions (i.e., they are \( \perp \) to one another)

\[ \vec{A} = \hat{x} A_x + \hat{y} A_y + \hat{z} A_z \]

in general, \( \vec{A} = \sum_{i=1}^{3} A_x \hat{A}_x \)

\( A_x \) is the projection of the vector \( \vec{A} \) onto \( x \) axis

looking into \( z-y \) plane (x unit of paper)

(\( z \) unit of paper)
Using the Pythagorean theorem (hypotenuse = sum of squares of 2 legs):

\[ A_r^2 = A_x^2 + A_y^2 \]

\[ |A|^2 = A_x^2 + A_y^2 + A_z^2 \]

\[ |A| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{\sum_{j=1}^{3} A_j^2} \]

The unit vector \( \hat{A} = \frac{A}{|A|} = \frac{\vec{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \) unit vector only represents direction.

We use the notation \( \vec{A} = (A_x, A_y, A_z) \) to represent the vector \( \vec{A} \).

Now, link at the distance between 2 arbitrary points in space \( P_1(x_1, y_1, z_1) \), \( P_2(x_2, y_2, z_2) \).

What is the dist between these 2 pts?

In order to figure out the distance, \( |R_{12}| \) we need to refresh our memory about some laws of vector addition.
vector equality:

2 vectors \( \vec{A}, \vec{B} \) are equal if and only if

1) magnitudes are equal \( |\vec{A}| = |\vec{B}| \)
2) directions are equal \( \hat{A} = \hat{B} \)
   \[ \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{B}}{|\vec{B}|} \]

BUT "equal" vectors need not be "identical"
since they may be displaced parallel in space.

vector addition

\[ \vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A} \] commutative prop. of addition

This results in the parallelogram rule or the head-to-tail rule:

\[ \vec{A} + \vec{B} \]

thus,

\[ \vec{C} = (x_A \vec{i} + y_A \vec{j} + z_A \vec{k}) + (x_B \vec{i} + y_B \vec{j} + z_B \vec{k}) \]
\[ = (x_A + x_B) \vec{i} + (y_A + y_B) \vec{j} + (z_A + z_B) \vec{k} \]
the components add just like scalars

unit
the same holds for subtraction

\[ \text{if } \vec{A} - \vec{B} = \hat{x}(A_x - B_x) + \hat{y}(A_y - B_y) + \hat{z}(A_z - B_z) \]

why is this important?
let us look at any arbitrary "vector" in space.

\[ \vec{R}_{12} = \vec{R}_2 - \vec{R}_1 \]

the distance vector from \( P_1 \rightarrow P_2 \)

the distance between \( P_1 \) and \( P_2 \)

\[ |\vec{R}_{12}| = \sqrt{(R_{2x}-R_{1x})^2 + (R_{2y}-R_{1y})^2 + (R_{2z}-R_{1z})^2} = R_{12} \]

\[ \vec{R}_{12} = \hat{x}(x_2-x_1) + \hat{y}(y_2-y_1) + \hat{z}(z_2-z_1) \]

\[ |\vec{R}_{12}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2} \quad \text{distance } P_1 P_2 \]

GPS

Applof this: the Global Position System (GPS)

uses this to determine absolute position in space — how?

principle of triangulation:
location \((x_0, y_0, z_0)\) can be of any object can be determined if we know distances \((d_1, d_2, d_3)\) between that object and 3 other independent known locations in space \((x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)\)
how do we do this:

G-P-S system is system of 24 satellites orbiting earth every 12 hrs at ~12k miles altitude plus 5 ground stations and the void maintaining their position
- satellite continuously emit coded time signals ref to atomic clock.

distance measured by measuring time for signal to go from satellite to "user" receiver.

\[ d \times t = \frac{c \times t}{2} \text{ time to go from satellite to receiver.} \]

essentially then

\[ d_1^2 = c^2 t_1^2 = (x_1-x_0)^2 + (y_1-y_0)^2 + (z_1-z_0)^2 \]
\[ d_2^2 = c^2 t_2^2 = (x_2-x_0)^2 + (y_2-y_0)^2 + (z_2-z_0)^2 \]
\[ d_3^2 = c^2 t_3^2 = (x_3-x_0)^2 + (y_3-y_0)^2 + (z_3-z_0)^2 \]

we have \( x_0, y_0, z_0, t_1, t_2, t_3 \)

unknown \( x_0, y_0, z_0 \) — 3 eqns, 3 unknowns — solve \( \| \)

(in reality — a bit more complicated — there is time

\( \) and that we need to measure — so for open, \( \| \) unknown

accuracy \( \approx 1m \)
let's now look at vector multiplication - 3 types

1) simple - scalar × vector \( \overrightarrow{B} = k \overrightarrow{A} \)
   - scalar constant
   \( = x(k\overrightarrow{A}_x) + y(k\overrightarrow{A}_y) + z(k\overrightarrow{A}_z) \)

2) **scalar product (dot product)** a **scalar quantity**

\[ \overrightarrow{A} \cdot \overrightarrow{B} = (\overrightarrow{A}) (\overrightarrow{B}) \cos \theta_{AB} \]

\( \theta_{AB} = \arccos \left( \frac{\overrightarrow{A} \cdot \overrightarrow{B}}{|\overrightarrow{A}| |\overrightarrow{B}|} \right) \)

\( |\overrightarrow{A} - \overrightarrow{B}| \leq |\overrightarrow{A}| |\overrightarrow{B}| \)

since \( \cos \leq 1 \)

- if \( 0 < \theta_{AB} < 90^\circ \), product > 0
- if \( 90^\circ < \theta_{AB} < 180^\circ \), product < 0

special cases:
- if \( \overrightarrow{A} \perp \overrightarrow{B} \), \( \theta_{AB} = 90^\circ \rightarrow \overrightarrow{A} \cdot \overrightarrow{B} = 0 \)
- if \( \overrightarrow{A} || \overrightarrow{B} \), \( \theta_{AB} = 0 \rightarrow \overrightarrow{A} \cdot \overrightarrow{B} = |\overrightarrow{A}| |\overrightarrow{B}| \)

note also that
\( |\overrightarrow{A}| = \sqrt{\overrightarrow{A} \cdot \overrightarrow{A}} \)

since orthogonal vectors' dot prod = 0 then
\[ \begin{bmatrix} x-y = x-\hat{x} = y-\hat{y} = 0 \\ x-x = y-y = z-z = 1 \end{bmatrix} \]

\[ \overrightarrow{A} \cdot \overrightarrow{B} = (x\overrightarrow{A}_x + y\overrightarrow{A}_y + z\overrightarrow{A}_z) \cdot (x\overrightarrow{B}_x + y\overrightarrow{B}_y + z\overrightarrow{B}_z) \]
\[ = A_x B_x + A_y B_y + A_z B_z \]

\( \) dot product is commutative \( \overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{B} \cdot \overrightarrow{A} \)

and is distributive \( \overrightarrow{A} \cdot (\overrightarrow{B} + \overrightarrow{C}) = \overrightarrow{A} \cdot \overrightarrow{B} + \overrightarrow{A} \cdot \overrightarrow{C} \)
3) **Vector Product (Cross Product)** — a vector quantity

\[ \vec{A} \times \vec{B} = \hat{n} |\vec{A}| |\vec{B}| \sin \theta_{AB}, \quad \hat{n} \text{ is a unit vector} \]

**Right Hand Rule**

- **Rule:** rotate \( \vec{A} \) towards \( \vec{B} \) (thumb and \( \vec{A} \rightarrow \vec{B} \)) -
- \( \hat{n} \) points in direction of thumb (right hand).

\[ \hat{n} \perp \vec{A}, \perp \vec{B} \]

**Note:** \( |\vec{A} \times \vec{B}| = \text{area of parallelogram} \)

\( \theta_{AB} \) is the included \( \theta \) between \( \vec{A} \) and \( \vec{B} \)

**Note:** "cross product" is **anti-commutative**

\[ \text{while dot product} \]

\[ \text{ie} \quad \vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \]

I leave it to you to show that:

- **Cross-product is distributive:**
  \[ \text{ie} \quad \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \]

and \( \vec{A} \times \vec{A} = 0 \) \( \text{note: } \theta_{AA} = 0 \)

**Note:** orthogonal unit vectors remain to one another in cross product

\[ \text{ie} \quad \hat{x} \times \hat{y} = \hat{z}, \quad \hat{y} \times \hat{z} = \hat{x}, \quad \hat{z} \times \hat{x} = \hat{y} \quad |\text{cyclic} | \quad \text{Note} \]

\[ \text{and} \]
we can express $\vec{A} \times \vec{B}$ in determinant form.

Remembering that $\hat{x} \times \hat{y} = \hat{z},\ \hat{y} \times \hat{z} = \hat{x},\ \hat{z} \times \hat{x} = \hat{y}$.

\[
\vec{A} \times \vec{B} = (\hat{x}A_x + \hat{y}A_y + \hat{z}A_z) \times (\hat{x}B_x + \hat{y}B_y + \hat{z}B_z)
\]

\[
= \hat{x}(\hat{y}A_yB_z - \hat{z}A_zB_y) + \hat{y}(\hat{z}A_zB_x - \hat{x}A_xB_z) + \hat{z}(\hat{x}A_xB_y - \hat{y}A_yB_x)
\]

\[
= \begin{vmatrix}
\hat{x} & \hat{y} & \hat{z} \\
A_x & A_y & A_z \\
B_x & B_y & B_z
\end{vmatrix}
\]

- Determinants

Now look at "TRIPLE PRODUCTS" —
- Two types:
  1) a scalar
     - Triple product: $\vec{A} \cdot (\vec{B} \times \vec{C})$
  2) a vector
     - Triple product: $\vec{A} \times (\vec{B} \times \vec{C})$
1) scalar triple product: (obey cyclic rule)
\[ \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) \]
leaves it to you to show:

\[ \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \]

2) vector triple product: \[ \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \neq (\mathbf{A} \times \mathbf{B}) \times \mathbf{C} \]

does not obey any simple law

the order of non-product multiplications is important

I leave it to you to evaluate \( \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \)

now we will review some of lab #1 (must finished)

where we discuss orthogonal coordinate systems —

an "orthogonal coordinate system" is one in which the three coordinates are mutually perpendicular.

i.e., like \( x, y, z \) —

there are however other useful systems:
cylindrical and spherical which are useful for measuring of quantities with cyl or spherical symmetry.
so so we will draw here 3 orthog word system, which are essential to be able to calculate quantities of length, area and volume.

In these 3 systems: 1) (Cartesian (rectilinear))
2) cylindrical
3) spherical

Note - for cyl. sph., the unit vectors defining the orthogonal directions are NOT independent of position, only in rect. system in that file.

* Table 3.1 (pg 110) in text gives the relationships of these 3 word systems.

Let's look at the one we have used so far:
1) Cartesian coordinate systems (rectilinear)
   \[ \hat{x}, \hat{y}, \hat{z} \] are the "base" or unit vectors, there are indep of position.

Diff. length: a vector
\[ \overrightarrow{dl} = \hat{x} dl_x + \hat{y} dl_y + \hat{z} dl_z \]

Diff. length along x axis
\[ \hat{x} \]

i.e. the diff. element \( dl \) is sum of the diff. components along the "base" direction.

unit
**NOTE**
If you need reference, what is the magnitude of a vector?

- **Surface Area**: Also a vector.
  - It has a magnitude equal to the product of 2 diffuse lengths AND a direction in a 3rd direction that is \( \perp \) to the plane of the 2 diffuse lengths. (normal to that plane)

\[ dS_x = \hat{x}dydz \quad \text{(\( \perp yz \) plane)} \]
\[ dS_y = \hat{y}dzdx \quad \text{(\( \perp zx \) plane)} \]
\[ dS_z = \hat{z}dxdy \quad \text{(\( \perp xy \) plane)} \]

**In general**: \[ dS^2 = \left| dS \right|^2 / a_{\perp} \] the normal \( \perp \) to plane of \( dS \).

This points away from the volume bounded by \( dS \) if it forms a volume.

\[ dS_x = \hat{x}dydz \]
Next Cartesian Coord. System

diff. volume: a scalar

not product of the 3 orthog. diff. lengths
\[ dV = dx dy dz \]

Now let's look at a cylindrical coord. system.

2) Cylindrical coordinate system

This system is useful when dealing with geometries that have cyl. symmetry (e.g., coaxial cables).

It is defined by the base vectors:

\[ \hat{r}, \hat{\phi}, \hat{z} \]

- \( \hat{r} \) rad. vector
- \( \hat{\phi} \) azimuthal vector
- \( \hat{z} \) z-axis as before.

The base vectors are

\[ \hat{r}, \hat{\phi}, \hat{z} \]

- Note that \( \phi \) depends upon "position".

As we move around circle of radius \( r \),

\( \hat{r} \) depends on \( \phi \).

Points in direction of increasing \( \phi \)
cyl. words
---
- base vectors \( \hat{r}, \hat{\theta}, \hat{z} \)
  - \( \hat{r} \) pts from origin along \( r \) (rad. coord.)
  - \( \hat{\theta} \) pts tangent to cyl. surf of rad. \( r \) in direction
    of increasing \( \theta \)
  - \( \hat{z} \) pts along \( z \)

NOTE: \( \hat{r} \) and \( \hat{\theta} \) both depend on point \( (r, \theta) \)
so not indep. as in cartesian.
BUT, still, angle behavior amongst the 3 coordinates:
\[
\hat{r} \times \hat{\theta} = \hat{z}, \quad \hat{\theta} \times \hat{z} = \hat{r}, \quad \hat{z} \times \hat{r} = \hat{\theta}
\]
since "orthog" \( \hat{r} \times \hat{r} = \hat{\theta} \times \hat{\theta} = \hat{z} \times \hat{z} = 0 \)

let's now do the length, area, volume:

\[
\text{diff length} \quad d\ell = r\,dr + \hat{\theta} \,dl_\theta + \hat{z} \,dz
\]

area length: \[
\int d\ell = \int r \,dr + \int (r \,dl_\theta) + \int \hat{z} \,dz
\]

now let's look at the diff area and then the volume:

a vector in cyl. words: \( \vec{A} = \hat{r}A_r + \hat{\theta}A_\theta + \hat{z}A_z \)

\[
|\vec{A}| = \sqrt{A_r^2 + A_\theta^2 + A_z^2}
\]
**Diff. surface area, \( \overrightarrow{dS} \) (again direction is \( \mathbf{\hat{n}} \) to surface)

\[
\overrightarrow{dS} = \overrightarrow{dS}_r + \overrightarrow{dS}_\theta + \overrightarrow{dS}_z \\
\overrightarrow{dS}_r = \mathbf{\hat{r}} \, dl_\phi \, dl_z = \mathbf{\hat{r}}(rd\chi \, dz) \\
\overrightarrow{dS}_\theta = \mathbf{\hat{\theta}} \, dl_z \, dl_r = \mathbf{\hat{\theta}}(dz \, dr) \\
\overrightarrow{dS}_z = \mathbf{\hat{z}} \, dl_r \, dl_\phi = \mathbf{\hat{z}}(dr \, rd\phi) \\
\overrightarrow{dS} = \mathbf{\hat{r}} \, rd\chi \, dz \\
\overrightarrow{dS} = \mathbf{\hat{\theta}} \, dz \, dr \\
\overrightarrow{dS} = \mathbf{\hat{z}} \, dr \, rd\phi \\
\overrightarrow{dS} = \mathbf{\hat{r}} \, rd\phi \, dz \]

**Diff. volume, \( \overrightarrow{dV} \)

\[
dV = dr \, rd\phi \, dl_z = dr(\phi \, dl_\phi) \, dz \]

**Note that if vector \( \vec{A} \) given as: \( \vec{A}(r, \phi, z) \)

Then \( |\vec{A}|^2 = r^2 A_r^2 + \phi^2 A_\phi^2 + z^2 A_z^2 \)

And \( |\vec{A}| = \sqrt{A_r^2 + A_\phi^2 + A_z^2} \)
Finally, we look at spherical words!
Here base vectors = \( \hat{R}, \hat{\theta}, \hat{\phi} \)
- \( R \) is measured from origin
- \( \theta \) is the zenith or from the z-axis
- \( \phi \) is the azimuthal or from the x-axis (just like in cyl words)

- \( \hat{R} \) directed outward from origin
- \( \hat{\theta} \) tan to sph. surf. \( \text{in rad} \). \( R \) - direction is that of increasing \( \theta \)
- \( \hat{\phi} \) tan to spherical direction, increasing \( \phi \)

\[
\rho = R \sin \theta \] same as \( r \) in spherical words.

Note: all unit vectors are positive only. \( \hat{R}, \hat{\theta}, \hat{\phi} \) so not maps
but they are still cyclic. i.e. \( \hat{R} \times \hat{\theta} = \hat{\phi}, \hat{\theta} \times \hat{\phi} = \hat{R}, \hat{\phi} \times \hat{R} = \hat{\theta} \)
Let's look at the 3 different quantities in spherical coordinates:

- **different length (a vector)**

\[ \vec{d}l = \hat{r} dr + \hat{\vartheta} d\vartheta + \hat{\varphi} d\varphi \]

\[ = \hat{r} dr + \hat{\vartheta} d\vartheta = \hat{\varphi} (r \sin \vartheta) d\varphi \]

\[ \Rightarrow \vec{d}l = \hat{r} dr + \hat{\vartheta} d\vartheta + \hat{\varphi} (r \sin \vartheta) d\varphi \]

- **area of a vector in spherical coordinates**

\[ \vec{A} = \hat{r} A_r + \hat{\vartheta} A_\vartheta + \hat{\varphi} A_\varphi \]

\[ |\vec{A}| = \sqrt{A_r^2 + A_\vartheta^2 + A_\varphi^2} \]

- **the different surface area - a vector**

\[ \vec{dS}_r = \hat{r} d\vartheta d\varphi \]

\[ = \hat{r} (r \sin \vartheta) d\vartheta d\varphi \]

\[ \vec{dS}_\vartheta = \hat{\vartheta} r \sin \vartheta d\varphi d\vartheta \]

\[ = \hat{\vartheta} (r \sin \vartheta) (d\varphi d\vartheta) \]

\[ \vec{dS}_\varphi = \hat{\varphi} r d\vartheta d\varphi \]

\[ = \hat{\varphi} (d\vartheta d\varphi) \]

\[ \Rightarrow \vec{dS}_\varphi = \hat{\varphi} r d\vartheta d\varphi \]

\[ \Rightarrow \vec{dS} = \vec{dS}_r + \vec{dS}_\vartheta + \vec{dS}_\varphi \]
An example of calculating surface area in spherical coordinates: consider a spherical strip of radius \( r \) bounded by angles \( \theta_1, \theta_2 \) as shown.

\[
dS = \hat{r} \, d\theta \, rd\phi
\]

\[
= \hat{r} (a \theta) (a \sin \theta d\theta)
\]

\[
\therefore S = \hat{r} \int dS = \hat{r} \int_{\theta_1}^{\theta_2} (a \sin \theta) a \theta d\theta
\]

\[
S = a^2 \left[ -a \cos \theta \right]_{\theta_1}^{\theta_2} = a^2 \left[ \sin \theta \right]_{\theta_1}^{\theta_2} \pi
\]

\[
S = a^2 \pi (\sin \theta_2 - \sin \theta_1)
\]

This trick is to carefully draw the geometry so you know what the differences are.
finally the diff volume is 

\[ dV = d\rho \, d\phi \, d\theta \]

\[ = (d\rho)(d\phi)(d\theta) \]

\[ dV = \rho^2 \sin \theta \, d\rho \, d\phi \, d\theta \]

now we have just gone thru 3 orthogonal and inter systems, what are the relationships between them? (part of lab 1 exercise)

rect \rightarrow spherical

\[ x, y, z \rightarrow \rho, \theta, \phi \]

\[ R = \sqrt{x^2 + y^2 + z^2} \]

\[ \theta = \arctan \left( \frac{\sqrt{x^2 + y^2}}{z} \right) \]

\[ \phi = \arctan \left( \frac{y}{x} \right) \]

Similarly

\[ x = (\rho \sin \theta) \cos \phi \]

\[ y = (\rho \sin \theta) \sin \phi \]

\[ z = \rho \cos \theta \]

the transformation between diff curv systems is given in table 3.2 / pg 117

leave it to student to do rect \rightarrow cyl / next we will look at diff operators
ASIDE

Example: distance between 2 pts in cyl. coords.

\[ r = \sqrt{x^2 + y^2} \]
\[ x = r \cos \phi, \quad y = r \sin \phi \]
\[ \phi = \arctan \left( \frac{y}{x} \right) \]

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \]

Rect.:

Cyl.:

\[ d = \sqrt{(x_2 \cos \phi_2 - x_1 \cos \phi_1)^2 + (x_2 \sin \phi_2 - x_1 \sin \phi_1)^2 + (z_2 - z_1)^2} \]

and we will do same thing for spherical coord.