1/11/05 Lect. #3 EE 135

Remainder:
Homework #1: chap. 1
Probs. 1.3, 1.8, 1.12, 1.16, 1.22 (all)
Due on Thu 1/13/05

New readings:
Chap. 2, pg. 35-58 (no new homework)

Need to get a 3rd lab section // urgent

Cannot have over 14-16/lab
Need 6-7 students to shift from Thu lab to new time (+ time)
1 student from Wed lab

Possible times:
F morning, evening, M morning, F afternoon. Only 3 choices

Need shift from mainly Thu

Note:// students with permission notes will get less priority
If I have to choose
No lab this week; starts next week. Discuss this week.

Review of phasors from lect #2.

Simple RC circuits:

\[ R C \frac{d}{dt} i(t) + i(t) = v_0(t) \]

Use loop eqn:

Writing phasor as \( \tilde{i}(t) = \Re \left[ \tilde{v} e^{j\omega t} \right] \) where \( \tilde{v} \) can be complex.

With unit the phasor (freq domain),

We get:

\[ \Re \left[ \tilde{v} e^{j\omega t} + \tilde{v} e^{j\omega t} \right] = \Re \left[ \tilde{v} e^{j\omega t} \right] \]

\[ L = v_0 e^{j(\omega - \pi/2)} \]

Remember \( e^{\jmath \theta} = \cos \theta + \jmath \sin \theta \)
\[ \hat{I}(R + \frac{j}{j\omega L}) = \hat{V}_s \]

- **Phasor Eqn (Impedance)**

or letting

\[ \hat{Z} = R + \frac{j}{j\omega C} \]

be the impedance phasor,

we get \[ \hat{I} \cdot \hat{Z} = \hat{V}_s \]

goes to lecture 2

**Complex Phasor Impedance**

Similarly we can now do the RL circuit

**Lmpt Eqn:**

\[ R\hat{i} + L \frac{d\hat{i}}{dt} = \hat{N}_s = V_0 \cos(\omega t + \phi - \frac{\pi}{2}) \]

\[ \hat{V}_s = V_0 e^{j(\phi - \frac{\pi}{2})} \]

\[ \lambda(t) = \text{Re}[\hat{I} e^{j\omega t}] \]

\[ \frac{d\lambda}{dt} = \text{Re}[j\omega \hat{I} e^{j\omega t}] \]

\[ R \cdot \text{Re}[\hat{I} e^{j\omega t}] + L \cdot \text{Re}[j\omega \hat{I} e^{j\omega t}] = \text{Re}[\hat{V}_s e^{j\omega t}] \]

where \( \hat{V}_s = V_0 e^{j(\phi - \frac{\pi}{2})} \)

\[ R \text{Re}[\hat{I} e^{j\omega t}] + L \text{Re}[j\omega \hat{I} e^{j\omega t}] = \text{Re}[\hat{V}_s e^{j\omega t}] \]

\[ \text{Re}[\hat{R} \hat{I} e^{j\omega t}] + \text{Re}[j\omega L \hat{I} e^{j\omega t}] = \text{Re}[\hat{V}_s e^{j\omega t}] \]

\[ \hat{R} \hat{I} + j\omega L \hat{I} = \hat{V}_s \]

\[ \hat{Z} = R + j\omega L \]

**Complex Impedance**

\[ \frac{\hat{I}}{\hat{Z}} = \frac{\hat{V}_s}{(R + j\omega L)} \]
3 \text{ Unit Lecture \#3.} \text{ Lecture \#4.} \text{ Ms. delay has left.}

\begin{align*}
\text{general procedure for finding } A(t) \text{ from } v(t) : \\
\frac{d}{dt} A(t) \quad \frac{1}{jw} \tilde{A} \\
\text{freq band} \quad \int A(t) dt \\
\text{imp. } A(w) \\
\text{freq domain} \quad \tilde{A} \quad \tilde{A} e^{jw} \\
\text{time domain} \quad A(t) \quad A \ln(w + i\epsilon)
\end{align*}

I leave it to you to work out general arrangement/mastery of phasor concepts results in many simpler formulae.

We will now use this info in our next discussion regarding transmission line.

\textit{Here is where we make judge between circuit theory and EM theory}

\textbf{*} Here we work with EM signals as if they were waves.

and define a transmission line as:

all structures 3 media which have to transfer energy or fluid. Information between two points
Such transmission lines include:
- nerve fibers in body
- telephone wires
- coaxial cables for TV or data transmission
- optical fibers, etc.

And we need to consider the wires connecting the circuit elements to the other as none than just transmission lines, i.e., we cannot ignore the wires in understanding how the circuit works, in particular cases.

Where the time it takes a signal (e.g., to propogate from input to output) is non-negligible compared to the period of the signal propagating.

\[ V_0(t) \rightarrow V_b(t) \]

\[ V_0(t) = V_{0,0} e^{iwt} \]

\[ f = \frac{1}{f_{\text{period}}} \]

The signal that reaches point B has been delayed by a time

\[ t = \frac{d}{c} \text{ time it takes elec signal to get from A to B} \]

If there are no losses in the line AB, then

\[ V_{AB}(t) = V_{AB}(t - \frac{d}{c}) \]

i.e., at B, the signal at time \( t \) is what it was at A at time \( \frac{d}{c} \) earlier.
\[ V_{BB^1}(t) = V_{BB^1}(t - \frac{1}{c}) = V_0 \cos \left( \omega \left( t - \frac{L}{c} \right) \right) \]

\[ = V_0 \cos \left( \omega t - \frac{\omega L}{c} \right) \]

\[ \Rightarrow \text{a "phase lag" of } \frac{\omega L}{c} \]

Example: let's suppose

at \( t = 0 \) \( V_{BB^1}(0) = V_0 \cos 0 = V_0 \)

assume signal freq \( \omega f = 1 \text{kHz} = 10^3 \text{rad/sec} \)

\( \omega = 2\pi f = 6.28 \times 10^3 \text{rad/sec} \)

\[ \therefore V_{BB^1}(0) = V_{BB^1}(0) \cos \left( -\frac{\omega L}{c} \right) \]

Remember units

if \( L = 10 \text{cm} \), \( \frac{L}{c} = \frac{10^{-1} \text{m}}{3 \times 10^8 \text{m/sec}} = \frac{1}{3} \times 10^{-9} \text{sec} \)

\[ \text{in a 10cm wire} \]

Remember argument in radians!

\[ \frac{\omega L}{c} = 6.28 \times 10^3 \times \frac{1}{3} \times 10^{-9} \text{sec} = \frac{2\pi}{3} \times 10^{-6} \text{ phase delay in radian} \]

\[ \therefore V_{BB^1}(0) = V_0 \cos \left( \frac{2\pi}{3} \times 10^{-6} \right) \approx 1.2 \times 10^{-4} \text{ deg.} \]

\[ \approx V_0 \cos (2.0944 \times 10^{-6}) \approx 1 \text{ to 3 decimal places} \]

\[ \therefore \text{we don't need to consider effect of the line on this "circuit".} \]

But suppose \( L = 10 \text{Km} / \text{a phone line} \)

then \( \frac{\omega L}{c} = \frac{6.28 \times 10^3 \text{rad/sec} \times 10^4 \text{m}}{3 \times 10^8 \text{m/sec}} = \frac{1}{3} \times 10^{-4} \text{ sec} \)

\[ \frac{\omega L}{c} = 2.0944 \times 10^{-1} \text{ rad} \approx 112^\circ \]

\[ V_{BB^1}(0) = V_0 \cos (2.0944) = 0.978 V_0 \text{ (a 2\% change)} \]
If the signal freq were 10x higher \( f = 10 \text{ kHz} \)

then \( \frac{\omega L}{C} = 2.0944 \) !

\[ \text{Phase} = 110.002^\circ \] definitely not !

\[ -5.009 \] definitely not !

\( V_{BB'}(D) = V_0 \cos(2.0944) = V_0(\text{negative}) \) ! a huge diff

Thus the value of the phase lag \( \frac{\omega L}{C} \) is important in
determining when we need to deal with the connectivity
wires between circuit elements. If thus a “transmission”
line can result in a phase shift much like a
so when can we ignore transmission line effects? Inductance can!

\[ \frac{\omega L}{C} = \frac{2\pi fL}{c} \]

Remember that \( C = \frac{1}{L} \lambda \): 

\[ \frac{f}{c} = \frac{1}{\lambda} \]

\[ \therefore \frac{\omega L}{c} = \frac{2\pi fL}{\lambda} \]

So if \( \frac{L}{\lambda} \) small we probably don’t need to worry.

Rule of thumb: How small? Magnitude of \( \frac{L}{\lambda} \geq 10^{-2} \) we probably

need to consider transmission effects.

\[ \Rightarrow \frac{\omega L}{c} \geq 10^{-2} \times 2\pi = 0.628 \text{ radians} = 36^\circ \text{ phase} \]

What this means is higher probability of
reflected waves looping back by load to generator.

NOTE: for very high yields, the \( L \) where this becomes
impossible can be quite small!
Example:

At telecommunication freqs — say \( f = 10^6 \text{Hz} \)

\[
\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^6} = 0.03 \text{m} = 3 \text{cm}
\]

\[
\therefore \text{If } \frac{L}{\lambda} > 10^{-2} \quad \text{to need to consider transmission line effect}
\]

\[
\text{then } L > 10^{-2} \times 3 \times 10^{-2} \text{m} = 3 \times 10^{-4} \text{m} = 300 \mu \text{m}
\]

(~10x the width of a human hair) / so in fibs need to deal with transmission line effect

In material, the phase velocity \( U_p \) is less than in air

for some materials can be 10x less! ~ even smaller \( L \) will result in phase delay!

\[
\text{In air (vac)} \quad c = 3 \times 10^8 \text{m/s}
\]

speed at which light/EM waves travel

\[
\frac{1}{c} = \frac{1}{3 \times 10^8} \text{m/s} \approx \frac{333}{12} \text{miles/second} \approx 110 \text{ft/s}
\]

In addition, in real materials, we need to consider "dispersion" as well, i.e. wave velocity

\( U_p \) is NOT constant with freq! thus, signals can get distorted.

\[
\text{distortionless} \quad \Downarrow \quad \text{up=down}
\]

\[
\text{distortion} \quad \Downarrow \quad \text{up=down} \quad \text{or worse}
\]
so what other effects might we need to consider when we have to take into account "real" wires in a circuit:

1. phase shift associated with time delay of signal
   { can give destructive interference between the 3 replica signals }

2. power law as the line - i.e. we have neglected any kind of "losses" so far

3. dispersion: \( U_p = U_p(w) \) wave vel depends on freq!
   remember, by Fourier

\[
\sum \frac{1}{n} \Rightarrow (\sum) \sqrt{n}
\]

a square pulse is sum of many sinusoidal waves of diff.

thus - if \( U_p = U_p(w) \) it means different components travel with different velocities - so at a given pt

on time down the line, the square pulse is distorted.

degree of distortion gets worse the longer the line.

\[
\begin{align*}
\text{\textbullet} & \quad \text{\textbullet} \\
\text{\textbullet} & \quad \text{\textbullet} \quad \text{no dispersion} \\
\text{\textbullet} & \quad \text{\textbullet} \quad \text{short line, dispersion} \\
\text{\textbullet} & \quad \text{\textbullet} \quad \text{longer line, dispersion}
\end{align*}
\]

so this week's next we will be dealing with transmission line -

they can come in several different types and can be classified

into different propagating modes

we will only consider 1 specific type of propagating

mode in this course - the transverse EM mode (TEM)

the electric fields are \perp\ directions to propagation
example of a TEM transmission line (usually depends on the geometry)

1. coaxial line

2. two wire line

3. parallel plate line

other geometries / the p/t to note is that we can calculate the electro/mag fields produced by these geometries once we start an an fig signal with them. we will learn that in the same class on 7/
So now let's look at how we can analyze circuits
when we need to take into account the inductance as 
"transmission lines".

Since trans. lines can produce a signal phase shift,
we can treat them as a combination of "lumped"
inductors, capacitors, etc.

Since there are diff. geometries for trans. lines,
we will consider the simplest geometry when and at them
as circuit elements:

we reg a transmission line as 2 parallel wires.
An analysis will be indep. of exact geometry.
we will develop a model called "the lumped element" model
here we represent a length of line as a
"lumped circuit" that has resistance, inductance,
capacitance and conductance characteristics.

Each diff. element of length of the line (Δz)
is represented by an equivalent lumped circuit,

\[ R'Δz \quad L'Δz \quad G'Δz \quad C'Δz \]

where

\[ R' = \text{equiv. resistance of the 2 wires/unit length in } \Omega/\text{m} \]
\[ L' = \text{equiv. inductance/unit length in } \text{H/m} \]
\[ G' = \text{equiv. conductance of insulation between 2 wires/unit length in } \text{S/m} \]
\[ C' = \text{equiv. capacitance/unit length in } \text{F/m} \]
so a length of transmission would be represented as

\[ \Delta z - \Delta z - \Delta z \]

\[ \Delta z - \Delta z \]

\[ \downarrow \]

\[ G' \Delta z \quad G' \Delta z \quad G' \Delta z \]

\[ c' \Delta z \quad c' \Delta z \quad c' \Delta z \]

\[ \Delta z - \Delta z \]

regardless of its shape -

the R', l', C', C' would be different for different materials and geometries

**Note:**

The text gives the values in these elements for different geometries, but in the name you will have a chance to derive them.

so for example, for coaxial: \[ R' = \frac{R_s}{2\pi} \left( \frac{1}{\alpha} + \frac{1}{b} \right) \]

where \[ R_s = \sqrt{\frac{\mu_0 \varepsilon_0}{2}} \]

- signal freq
- permeability of the wire
- \( \mu_0 \) = vacuum permeability
- \( \varepsilon_0 \) = permittivity of free space

\[ \alpha = \frac{\text{radius of outer conductor}}{\text{radius of inner conductor}} \]

\[ b = \text{inner rad of outer cond.} \]

thus when \( R' \) goes up as freq goes up,

\[ R' \to 0 \text{ for perfect conductor since } \varepsilon \to \infty \]

Last
eventually you will be able to work out all the
sigs in the diff geometries but what you will see
is this: [I give the coaxial line as an example]

\[ L' = \frac{M}{Z'} \ln(b/a) \quad M = \text{permeability of shell}
\]
\[ \sigma' = Z' / \ln(b/a) \quad \sigma = \text{permeivity of shell}
\]

\[ L' \sigma' = \frac{M \ln(b/a) \sigma' \ln(b/a)}{Z'} = ME \quad ME = L' \sigma' \]

this holds for all geometries

note that from before the phase vel (wavevel)
of the liquid is \( U_p = \sqrt{\frac{1}{\sigma'}} \quad \text{Maxwell} \)

\[ \text{so } L' \sigma' = \frac{1}{U_p^2} \]

similarly we will find that

\[ \frac{G'}{C'} = \frac{\sigma}{\varepsilon} \]

depends on material

if insulator vs air a vacuum
\( \varepsilon = \varepsilon_0, \mu = \mu_0, \sigma = 0 \rightarrow G' = 0 \)

which makes sense -

the bell's, the insulation, the bus

"leakage" from 1 line to the other