In the process of analyzing an electronic circuit, a series of linear equations will be generated and will need to be solved simultaneously. Suppose the following set of three equations is obtained for three currents (I₁, I₂, and I₃):

1. \(2I₁ + I₂ - I₃ = 3\)
2. \(I₁ + 2I₂ - 3I₃ = 7\)
3. \(I₁ - I₂ - I₃ = -1\)

\[\begin{align*}
3I₁ - 2I₂ &= 2 \\
3I₁ - 5I₃ &= 5 \\
-7I₃ &= 7
\end{align*}\]

1. Find I₁, I₂, and I₃. Use any technique you’d like. All work must be shown.

When energy storage elements (like capacitors and inductors) are introduced into a circuit, the linear equations become differential equations, and the solution becomes time dependent. As a result, we will consider first and second order differential equations later in the course.

Consider the following first-order differential equation for a function \(v(t)\).

\[\frac{dv}{dt} + 2v = 5\]

2a. The solution of \(v(t)\) consists of two parts: a homogeneous solution and a particular solution. What form does each of these solutions take?

2b. Prove that your solution in 2a. is valid by substituting your trial solution into the differential equation, and verifying you have a true statement.

3a. Assume further that you know that \(v(0) = 8\). Use this initial condition to determine the exact solution for \(v(t)\).

3b. Sketch \(v(t)\) for values of \(t > 0\).
Consider the following second-order differential equation:

\[ \frac{d^2 v}{dt^2} + 8 \frac{dv}{dt} + 12v = 0 \]

4a. Using a trial solution of the form \( v(t) = Ke^{st} \), substitute this result into the differential equation and determine the two possible solutions for \( s \).

4b. What form should the solution for \( v(t) \) take?

\[ v(t) = k_1 e^{-6t} + k_2 e^{-2t} \]  
where \( k_1 + k_2 = \text{const} \)

In more complicated circuits, it will be easier to work in the frequency domain instead of the time domain. This will allow us to convert complicated differential equations (in time) to linear equations (in frequency). In order to do the transformations, a strong knowledge of complex number manipulations in necessary.

Given two complex numbers: \( Z_1 = -4 + j \) and \( Z_2 = 2 - 2j \), find the following expressions:

5a. \( Z_1 - Z_2 \):

\[ -4 + j - (2 - 2j) \rightarrow -6 + 3j \]

5b. \( Z_1 Z_2 \):

\[ (-4 + j)(2 - 2j) \rightarrow -8 + 2j + 8j - 2j^2 \rightarrow -6 + 10j \]

6. Express \( Z_2 \) in polar form, i.e., \( Z_2 \) can be written in the form \( |Z_2| e^{i\theta} \). Find \( |Z_2| \) and \( \theta \).

7. Simplify:

\[ \frac{-1 + j}{\sqrt{7} e^{-180^\circ} j} \]

\[ \frac{\sqrt{7} e^{180^\circ}}{\sqrt{7} e^{-180^\circ} j} \]

\[ \frac{-1 + j}{\sqrt{7} (-1 + j 0)} \]

\[ \frac{-1 + j}{-\sqrt{7}} \]

\[ \frac{\sqrt{7} - \sqrt{7} j}{7} \]

\[ \text{polar answer is also ok} \]