Lecture 3
Circuit Laws,
Voltage & Current Dividers
KIRCHHOFF’S CURRENT LAW

• The net current entering a node is zero.

• Alternatively, the sum of the currents entering a node equals the sum of the currents leaving a node.
Figure 1.20 Elements $A$, $B$, and $C$ are connected in series.
KIRCHHOFF’S VOLTAGE LAW

The algebraic sum of the voltages equals zero for any closed path (loop) in an electrical circuit.
Figure 1.27 For this circuit, we can show that $v_a = v_b = v_c = -v_c$. Thus the magnitudes and actual polarities of all three voltages are the same.
Using KVL, KCL, and Ohm’s Law to Solve a Circuit

![Circuit Diagram]

- Voltage source $V_s$
- Current $i_x$ through a 10 Ohm resistor
- Current $i_y$ leaving a node
- Voltage $v_x$
- Current $0.5i_x$ through a 5 Ohm resistor
- 15 V voltage source
\[ i_y = \frac{15 \text{ V}}{5 \text{ Ω}} = 3 \text{ A} \]

\[ i_x + 0.5i_x = i_y \]

\[ i_x = 2 \text{ A} \]
\[ v_x = 10i_x = 20 \text{ V} \]

\[ V_s = v_x + 15 \]

\[ V_s = 35 \text{ V} \]
Chapter 2
Resistive Circuits

1. Solve circuits (i.e., find currents and voltages of interest) by combining resistances in series and parallel.

2. Apply the voltage-division and current-division principles.

3. Solve circuits by the node-voltage technique.
4. Solve circuits by the mesh-current technique.

5. Find Thévenin and Norton equivalents and apply source transformations.

6. Apply the superposition principle.

7. Draw the circuit diagram and state the principles of operation for the Wheatstone bridge.
Figure 2.1 Series resistances can be combined into an equivalent resistance.

\[ v = v_1 + v_2 + v_3 = iR_1 + iR_2 + iR_3 = i(R_1 + R_2 + R_3) = iR_{eq} \]
Figure 2.2 Parallel resistances can be combined into an equivalent resistance.

\[
i = i_1 + i_2 + i_3 = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} = v \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{v}{R_{eq}}
\]
Figure 2.3 Resistive network for Example 2.1.
Figure 2.4 Resistive networks for Exercise 2.1.

(a) $R_1 = 2 \Omega$, $R_2 = 6 \Omega$, $R_3 = 3 \Omega$, $R_4 = 2 \Omega$

(b) $R_1 = 10 \Omega$, $R_2 = 6 \Omega$, $R_3 = 3 \Omega$, $R_4 = 8 \Omega$

(c) $R_1 = 1 \text{k}\Omega$, $R_2 = 2 \text{k}\Omega$, $R_3 = 3 \text{k}\Omega$

(d) $R_1 = 100 \Omega$, $R_2 = 50 \Omega$
\[ R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{1}{\frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6}} = \frac{1}{\frac{11}{12}} = \frac{12}{11} \]

Diagram:
- \( R_1 = 2 \, \Omega \)
- \( R_2 = 3 \, \Omega \)
- \( R_3 = 6 \, \Omega \)
- \( R_4 = 6 \, \Omega \)
\[ R_{total} = R_1 + 1 \Omega = 3 \Omega \]
\[
\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}
\]

\[
R_{eq} = 2.0 \Omega
\]
\[ R_{eq} = 8\Omega + 2\Omega = 10\Omega \]
\[
\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} \Omega \rightarrow R_{eq} = 5\Omega
\]
(d) $R_1 = 1 \text{k}\Omega$

$R_3 = 3 \text{k}\Omega$

$R_2 = 2 \text{k}\Omega$
Circuit Analysis using Series/Parallel Equivalents

1. Begin by locating a combination of resistances that are in series or parallel. Often the place to start is farthest from the source.

2. Redraw the circuit with the equivalent resistance for the combination found in step 1.
3. Repeat steps 1 and 2 until the circuit is reduced as far as possible. Often (but not always) we end up with a single source and a single resistance.

4. Solve for the currents and voltages in the final equivalent circuit.
(a) Original circuit

(b) Circuit after replacing $R_2$ and $R_3$ by their equivalent

(c) Circuit after replacing $R_1$ and $R_{eq1}$ by their equivalent

**Figure 2.5** A circuit and its simplified versions. See Example 2.2.
Figure 2.6 After reducing the circuit to a source and an equivalent resistance, we solve the simplified circuit. Then, we transfer results back to the original circuit. Notice that the logical flow in solving for currents and voltages starts from the simplified circuit in (c).
\[ i_2 = \frac{v_2}{R_2} = \frac{60V}{30\Omega} = 2A \]

\[ i_3 = \frac{v_2}{R_3} = \frac{60V}{60\Omega} = 1A \]

\[ v_1 = R_1 i_1 = (10\Omega)(3A) = 30V \]
Figure 2.7 Circuits for Exercise 2.2.
1/\( R_{eq} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{120} + \frac{1}{120} = \frac{1}{60} \)

\( R_{eq} = 120 \times 13 = 1560 \)

\( 1/17 = 30 \)

\( 26 \)

\( 10 \times 10 = 100 \)

\( 20 \times 10 = 100 \times 0.9 \times 10 = 1000 \times 0.9 = 900 \)

\( 20 \times 0.1 = 2 \times 0.9 = 18 \)

\( 2.4 \times 0.9 = 2.16 \times 0.9 = 1.944 \)

\( 10 \times 2 = 20 \)

\( 9 \times 10 = 90 \)

\( 9 \times 10 = 90 \)

\( 0.9 \times 0.9 = 0.81 \)

\( 9 \times 0.9 = 8.1 \)
Figure 2.8 Circuit used to derive the voltage-division principle.
Voltage Division

Of the total voltage, the fraction that appears across a given resistance in a series circuit is the ratio of the given resistance to the total series resistance.

\[ v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + R_3} v_{\text{total}} \]

\[ v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{\text{total}} \]
$R_1 = 1 \text{k}\Omega$

$R_2 = 1 \text{k}\Omega$

$R_3 = 2 \text{k}\Omega$

$R_4 = 6 \text{k}\Omega$

$v_{\text{total}} = 15 \text{V}$

$v_1 - \quad + \quad - v_4 +$

Figure 2.9 Circuit for Example 2.3.
Application of the Voltage-Division Principle

\[ v_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} \frac{v_{\text{total}}}{1000} = \frac{1000}{1000 + 1000 + 2000 + 6000} \times 15 = 1.5V \]
Figure 2.10 Circuit used to derive the current-division principle.

\[ R_{eq} = \frac{R_1R_2}{R_1 + R_2} \rightarrow i = i_{total} \]

\[ R_{eq} = \frac{R_1R_2}{R_1 + R_2} \]
For two resistances in parallel, the fraction of the total current flowing in a resistance is the ratio of the other resistance to the sum of the two resistances.

\[
\begin{align*}
    i_1 &= \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_{\text{total}} \\
    i_2 &= \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{\text{total}}
\end{align*}
\]
Find $i_3$

(a) Original circuit

(b) Equivalent circuit obtained by combining $R_2$ and $R_3$

\[ i_s = \frac{v_s}{R_1 + R_x} = \frac{100}{60 + 20} = 1.25 \, A \]

\[ i_3 = \frac{R_2}{R_2 + R_3} \cdot i_s = \frac{30}{30 + 60} \cdot (1.25) = 0.417 \, A \]
Find \( i_1 \)

(a) Original circuit

(b) Circuit after combining \( R_2 \) and \( R_3 \)

Figure 2.12 Circuit for Example 2.5.
Application of the Current-Division Principle

\[ R_{\text{eq}} = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20 \Omega \]

\[ i_1 = \frac{R_{\text{eq}}}{R_1 + R_{\text{eq}}} i_s = \frac{20}{10 + 20} \frac{15}{10} = 10 \text{A} \]
Use the voltage division principle to find the unknown voltages.
\[ v_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} \]
\[ v_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \]
\[ v_3 = \frac{R_3}{R_1 + R_2 + R_3 + R_4} \]
\[ v_4 = \frac{R_4}{R_1 + R_2 + R_3 + R_4} \]

\[ v_s = \frac{5}{5+10+15+30} (120V) = \frac{120V}{12} = 10V \]
\[ v_s = \frac{10}{5+10+15+30} (120V) = \frac{120V}{6} = 20V \]
\[ v_s = \frac{15}{5+10+15+30} (120V) = \frac{120V}{4} = 30V \]
\[ v_s = \frac{30}{5+10+15+30} (120V) = \frac{120V}{2} = 60V \]
\[ R_{eq} = \frac{1}{\frac{1}{7} + \frac{1}{5}} = 2.92\Omega \]

\[ v_1 = \frac{R_1}{R_1 + R_{eq} + R_4} \cdot v_s = \frac{3}{3 + 2.92 + 4} \cdot (20V) = \frac{(3)(20V)}{9.92} = 6.05V \]

\[ v_2 = \frac{R_{eq}}{R_1 + R_{eq} + R_4} \cdot v_s = \frac{2.92}{3 + 2.92 + 4} \cdot (20V) = \frac{(2.92)(20V)}{9.92} = 5.89V \]

\[ v_4 = \frac{R_4}{R_1 + R_{eq} + R_4} \cdot v_s = \frac{4}{3 + 2.92 + 4} \cdot (20V) = \frac{(4)(20V)}{9.92} = 8.06V \]
Use the current division principle to find the currents

Figure 2.15 Circuits for Exercise 2.4.
\[
\begin{align*}
  i_1 &= \frac{R_3}{R_1 + R_2 + R_3} \quad i_t = \frac{15}{10 + 20 + 15} \quad (3) = 1\, A \\
  i_3 &= \frac{R_1 + R_2}{R_1 + R_2 + R_3} \quad i_t = \frac{30}{10 + 20 + 15} \quad (3) = 2\, A
\end{align*}
\]
\[ R_{eq} = \frac{1}{\frac{1}{10} + \frac{1}{10}} = 5 \]

\[ i_1 = \frac{R_{eq}}{R_{eq} + R_1} i_s = \frac{5}{5 + 10} (3) = 1 A \]

\[ i_2 = \frac{R_{eq}}{R_{eq} + R_1} i_s = \frac{5}{5 + 10} (3) = 1 A \]

\[ i_3 = \frac{R_{eq}}{R_{eq} + R_1} i_s = \frac{5}{5 + 10} (3) = 1 A \]
Resistor Cube