digital form (a sequence of numbers) by an analog-to-digital converter (ADC), a digital computer uses the digitized input signal to compute a sequence of values for the output signal, and finally (if desired) the computed values are converted to analog form by a digital-to-analog converter (DAC) to produce the output signal \( y(t) \).

16. If a signal contains no components with frequencies higher than \( f_H \), the signal can be exactly reconstructed from its samples, provided that the sampling rate \( f_s \) is selected to be more than twice \( f_H \).

17. Approximately equivalent digital filters can be found for RLC filters.

### Problems

#### Section 6.1: Fourier Analysis, Filters, and Transfer Functions

**P6.1.** What is the fundamental concept of Fourier theory?

**P6.2.** The full-wave rectified cosine wave shown in Figure P6.2 can be written as

\[
v_{tw}(t) = \frac{2}{\pi} + \frac{4}{\pi(1)(3)} \cos(4000\pi t) - \frac{4}{\pi(3)(5)} \cos(8000\pi t) + \ldots + \frac{4(-1)^{(n/2+1)}}{\pi(n - 1)(n + 1)} \cos(2000n\pi t) + \ldots
\]

in which \( n \) assumes even integer values. Use a computer program of your choice to compute and plot the sum through \( n = 60 \) for \( 0 \leq t \leq 2 \) ms. Compare your plot with the waveform shown in Figure P6.2.

![Figure P6.2](image)

**P6.3.** The triangular waveform shown in Figure P6.3 can be written as the infinite sum:

\[
v(t) = 1 + \frac{8}{\pi^2} \cos(2000\pi t) + \frac{8}{(3\pi)^2} \cos(6000\pi t) + \ldots + \frac{8}{(n\pi)^2} \cos(2000n\pi t) + \ldots
\]

in which \( n \) takes odd integer values only. Use a computer program of your choice to compute and plot the sum through \( n = 19 \) for \( 0 \leq t \leq 2 \) ms. Compare your plot with the waveform shown in Figure P6.3.

![Figure P6.3](image)

**P6.4.** Fourier analysis can be used to show that the sawtooth waveform of Figure P6.4 can be

* Denotes that answers can be found on both CDs and on the web site www.prenhall.com/hambley
written as
\[ v_{st}(t) = \frac{1}{2} - \frac{1}{\pi} \sin(2000\pi t) \]
\[ - \frac{1}{2\pi} \sin(4000\pi t) - \frac{1}{3\pi} \sin(6000\pi t) \]
\[ - \cdots - \frac{1}{n\pi} \sin(2000n\pi t) - \cdots \]

Use a computer program of your choice to compute and plot the sum through \( n = 5 \) for \( 0 \leq t \leq 2 \) ms. Repeat for the sum through \( n = 10 \).

![Figure P6.4](image)

**P6.5.** The transfer function \( H(f) = V_{out}/V_{in} \) of a filter is shown in Figure P6.5. If the input signal is given by
\[ v_{in}(t) = 2\cos(10^4\pi t + 30^\circ) \]
find an expression (as a function of time) for the steady-state output of the filter.

*P6.6.** Repeat Problem P6.5 if the input voltage is given by
\[ v_{in}(t) = 2\cos(5000\pi t + 30^\circ) + 2\cos(15,000\pi t) \]

**P6.7.** Repeat Problem P6.5 if the input voltage is given by
\[ v_{in}(t) = 2\cos(10^4\pi t + 30^\circ) + 2\cos(30,000\pi t) \]

*P6.8.** The input to a certain filter is given by
\[ v_{in}(t) = 2\cos(10^4\pi t - 25^\circ) \]
and the steady-state output is given by
\[ v_{out}(t) = \cos(10^4\pi t + 20^\circ) \]

Determine the (complex) value of the transfer function of the filter for \( f = 5000 \) Hz.

**P6.9.** The sawtooth waveform of Problem P6.4 is applied as the input to a filter having the transfer function shown in Figure P6.9. Assume that the phase of the transfer function is \( \angle H(f) = 0 \). Determine the steady-state output of the filter.
**P6.10.** The triangular waveform of Problem P6.3 is the input for a filter having the transfer function shown in Figure P6.10. Assume that the phase of the transfer function is \( H(f) = 0 \). Determine the steady-state output of the filter.

![Figure P6.10](image)

**P6.11.** Consider a circuit for which the output voltage is the time derivative of the input voltage. This is illustrated in Figure P6.11. Assume that the input voltage is given by

\[ v_{in}(t) = V_{max} \cos(2\pi ft) \]

and find an expression for the output voltage as a function of time. Find an expression for the transfer function of the differentiator. Plot the magnitude and phase of the transfer function versus frequency.

![Figure P6.11](image)

**P6.12.** Consider a circuit for which the output voltage is the running-time integral of the input voltage. This is illustrated in Figure P6.12. Assume that the input voltage is given by

\[ v_{in}(t) = V_{max} \cos(2\pi ft) \]

and find an expression for the output voltage as a function of time. Find an expression for the transfer function of the integrator. Plot the magnitude and phase of the transfer function versus frequency.

![Figure P6.12](image)

**P6.13.** Consider a system for which the output voltage is

\[ v_o(t) = v_{in}(t) + v_{in}(t - 10^{-3}) \]

(The output equals the input plus the input delayed by 1 ms.) Assume that the input voltage is given by

\[ v_{in}(t) = V_{max} \cos(2\pi ft) \]

and find an expression for the output voltage as a function of time. Find an expression for the transfer function of the system. Use a computer program of your choice to plot the magnitude of the transfer function versus frequency for the range from 0 to 2000 Hz. Comment on the result.

**Section 6.2: First-Order Lowpass Filter**

**P6.14.** Draw the circuit diagram of a first-order \( RC \) lowpass filter and give the expression for the half-power frequency in terms of the circuit components. Sketch the magnitude and phase of the transfer function versus frequency.

**P6.15.** Repeat Problem P6.14 for an \( RL \) filter.

**P6.16.** Suppose that an input signal given by

\[ v_{in}(t) = 5 \cos(500\pi t) + 5 \cos(1000\pi t) + 5 \cos(2000\pi t) \]

is applied to the lowpass \( RC \) filter shown in Figure P6.16. Find an expression for the output signal.
A first-order \( RC \) lowpass filter with a half-power frequency of 10 kHz is needed. Determine the value of the capacitance if the resistance is 1 k\( \Omega \).

An input signal containing components that range in frequency from 100 Hz to 10 kHz is to be passed through a first-order lowpass filter. It is desired to reduce the amplitude of the 10-kHz component by a factor of 100. Determine the half-power frequency required for the filter. By what factor is a component at 1 kHz changed in amplitude in passing through this filter?

Consider a first-order \( RC \) lowpass filter. At what frequency (in terms of \( f_B \)) is the phase shift equal to \(-1^\circ\)? \(-10^\circ\)? \(-89^\circ\)?

Consider the circuit shown in Figure P6.20a. This circuit consists of a source having an internal resistance of \( R_s \), an \( RC \) lowpass filter, and a load resistance \( R_L \). a. Show that the transfer function of this circuit is given by

\[
H(f) = \frac{V_{out}}{V_s} = \frac{R_L}{R_s + R + R_L} \times \frac{1}{1 + j(f/f_B)}
\]

in which the half-power frequency \( f_B \) is given by

\[
f_B = \frac{1}{2\pi R_t C} \quad \text{where} \quad R_t = \frac{R_L(R_s + R)}{R_L + R_s + R}
\]

Notice that \( R_t \) is the parallel combination of \( R_L \) and \( (R_s + R) \). (Hint: One way to make this problem easier is to rearrange the circuit as shown in Figure P6.20b and then to find the Thévenin equivalent for the source and resistances.) b. Given that \( C = 1 \mu F, R_s = 1 \) k\( \Omega \), \( R = 2 \) k\( \Omega \), and \( R_L = 3 \) k\( \Omega \), sketch (or use a computer to plot) the magnitude of \( H(f) \) to scale versus frequency up to \( 3f_B \).

Derive an expression for the transfer function \( H(f) = \frac{V_{out}}{V_{in}} \) for the circuit shown in Figure P6.21. Find an expression for the half-power frequency. b. Given \( R_1 = 1 \) k\( \Omega \), \( R_2 = 1 \) k\( \Omega \), and \( L = 10 \) m\( H \), sketch the magnitude of the transfer function versus frequency.

Find Thévenin equivalent
P6.22. Sketch the magnitude of the transfer function \( H(f) = \frac{V_{\text{out}}}{V_{\text{in}}} \) to scale versus frequency for the circuit shown in Figure P6.22. What is the value of the half-power frequency? (*Hint: Start by finding the Thévenin equivalent circuit seen by the capacitance.*)

![Figure P6.22](image)

P6.23. A 2-V-rms 10-kHz sinusoidal input is applied to a first-order RC lowpass filter (see Figure 6.7 on page 254). The output voltage is 0.5 V rms. Determine the rms output voltage after the frequency of the input signal is raised to 50 kHz.

Section 6.3: Decibels, the Cascade Connection, and Logarithmic Frequency Scales

P6.24. What is a notch filter? What is one application?

P6.25. What is the advantage of converting transfer-function magnitudes to decibels before plotting?

P6.26. Two filters with transfer functions \( H_1(f) \) and \( H_2(f) \) are cascaded in the order 1–2. Give the expression for the overall transfer function of the cascade. Repeat if the transfer function magnitudes are expressed in decibels denoted as \( |H_1(f)|_{\text{dB}} \) and \( |H_2(f)|_{\text{dB}} \). What caution concerning \( H_1(f) \) must be considered?

P6.27. Suppose that \( |H(f)| = 0.5 \). Find the decibel equivalent. Repeat for \( |H(f)| = 2 \).

P6.28. a. Given \( |H(f)|_{\text{dB}} = -10 \text{ dB} \), find \( |H(f)| \).
   b. Repeat for \( |H(f)|_{\text{dB}} = 10 \text{ dB} \).

P6.29. a. What frequency is one octave higher than 500 Hz? b. Three octaves lower? c. Two decades higher? d. One decade lower?

P6.30. a. What frequency is halfway between 100 and 3000 Hz on a logarithmic frequency scale? b. On a linear frequency scale?

P6.31. a. How many decades are between \( f_1 = 100 \text{ Hz} \) and \( f_2 = 3.5 \text{ kHz} \)? b. How many octaves?

P6.32. Two first-order lowpass filters are in cascade as shown in Figure P6.32. The transfer functions are

\[
H_1(f) = H_2(f) = \frac{1}{1 + j(f/f_B)}
\]

a. Write an expression for the overall transfer function. b. Find an expression for the half-power frequency for the overall transfer function in terms of \( f_B \).

(Comment: This filter cannot be implemented by cascading two simple RC lowpass filters like the one shown in Figure 6.7 on page 254 because the transfer function of the first circuit is changed when the second is connected. Instead, a buffer amplifier, such as the voltage follower discussed on page 618, must be inserted between the RC filters.)

![Figure P6.32](image)
P6.33. Suppose that two filters are in cascade. At a given frequency \( f_1 \), the transfer function values are \( |H_1(f_1)|_dB = -20 \) and \( |H_2(f_1)|_dB = -10 \). Find the magnitude of the overall transfer function in decibels at \( f = f_1 \).

Section 6.4: Bode Plots

P6.34. What is a Bode plot?

P6.35. What is the slope of the high-frequency asymptote for the Bode magnitude plot for a first-order lowpass filter? The low-frequency asymptote? At what frequency do the asymptotes meet?

P6.36. Sketch the asymptotic Bode magnitude and phase plots to scale for the circuit shown in Figure P6.36.

![Figure P6.36](image)

P6.37. A transfer function is given by

\[
H(f) = \frac{100}{1 + j(f/1000)}
\]

Sketch the asymptotic magnitude and phase Bode plots to scale. What is the value of the half-power frequency?

P6.38. A transfer function is given by

\[
H(f) = \frac{10}{1 - j(f/500)}
\]

Sketch the asymptotic magnitude and phase Bode plots to scale. What is the value of the half-power frequency?

P6.39. Sketch the asymptotic magnitude and phase Bode plots to scale for the transfer function

\[
H(f) = \frac{1 - j(f/100)}{1 + j(f/100)}
\]

P6.40. In solving Problem P6.11, we find that the transfer function of a differentiator circuit is given by \( H(f) = j2\pi f \). Sketch the Bode magnitude and phase plots to scale. What is the slope of the magnitude plot?

*P6.41. In solving Problem P6.12, we find that the transfer function of an integrator circuit is given by

\[
H(f) = \frac{1}{j2\pi f}
\]

Sketch the Bode magnitude and phase plots to scale. What is the slope of the magnitude plot?

P6.42. Find the transfer function and draw the Bode magnitude and phase plots for the circuit shown in Figure P6.42.

![Figure P6.42](image)

P6.43. Find the transfer function and draw the asymptotic Bode magnitude and phase plots for the circuit shown in Figure P6.43.

![Figure P6.43](image)

Section 6.5: First-Order Highpass Filter

P6.44. Draw the circuit diagram of a first-order \( RC \) highpass filter and give the expression for the half-power frequency in terms of the circuit components.
**P.6.45.** What is the slope of the high-frequency asymptote for the Bode magnitude plot for a first-order highpass filter? The low-frequency asymptote? At what frequency do the asymptotes meet?

**P.6.46.** Consider the circuit shown in Figure P.6.46. Sketch the Bode magnitude and phase plots to scale for the transfer function $H(f) = V_{out}/V_{in}$.

![Figure P.6.46](image)

**P.6.47.** Consider the circuit shown in Figure P.6.47. Sketch the Bode magnitude and phase plots to scale for the transfer function $H(f) = V_{out}/V_{in}$.

![Figure P.6.47](image)

**P.6.48.** Consider the circuit shown in Figure P.6.48. Sketch the asymptotic Bode magnitude and phase plots to scale for the transfer function $H(f) = V_{out}/V_{in}$.

![Figure P.6.48](image)

**P.6.49.** A first-order highpass filter (such as Figure 6.19 on page 268) is required that attenuates a 60-Hz input component by 40 dB. What value is required for the break frequency of the filter? By how many dB is the 600-Hz component attenuated by this filter? If $R = 1 \text{ k}\Omega$, what is the value of $C$?

**P.6.50.** Consider the first-order highpass filter shown in Figure P.6.50. The input signal is given by $v_{in}(t) = 5 \cos(200\pi t) + 5 \cos(2000\pi t)$

Find an expression for the output $v_{out}(t)$ in steady-state conditions.

![Figure P.6.50](image)

* **P.6.51.** Repeat Problem P.6.50 if the input signal is given by

$$v_{in}(t) = 5 + 5 \cos(2000\pi t)$$

**Section 6.6: Series Resonance**

* **P.6.52.** Consider the series resonant circuit shown in Figure P.6.52 with $L = 20 \mu H$, $R = 14.14 \Omega$, and $C = 1000 \text{ pF}$. Compute the resonant frequency, the bandwidth, and the half-power frequencies. Assuming that the frequency of the source is the same as the resonant frequency, find the phasor voltages across the elements and sketch a phasor diagram.

![Figure P.6.52](image)
P6.53. Repeat Problem P6.52 for $L = 20 \, \mu H$, $R = 1.414 \, \Omega$, and $C = 1000 \, pF$.

P6.54. At the resonant frequency $f_0 = 1 \, MHz$, a series resonant circuit with $R = 50 \, \Omega$ has $|V_R| = 2 \, V$ and $|V_L| = 20 \, V$. Determine the values of $L$ and $C$. What is the value of $|V_C|$?

P6.55. A series resonant circuit has $B = 50 \, kHz$, $f_0 = 400 \, kHz$, and $R = 20 \, \Omega$. Determine the values of $L$ and $C$.

P6.56. A series resonant circuit has $f_0 = 10 \, MHz$ and $B = 200 \, kHz$. The minimum value of $|Z_s|$ is $10 \, \Omega$. Determine the values of $R$, $L$, and $C$.

Section 6.7: Parallel Resonance

P6.57. Determine the $L$ and $C$ values for Figure 6.29 on page 277, given $R = 2 \, k\Omega$, $f_0 = 10 \, MHz$, and $B = 200 \, kHz$. If $I = 10^{-3} \, \angle 0^\circ$, draw a phasor diagram showing the currents through each of the elements in the circuit at resonance.

P6.58. A parallel resonant circuit has $R = 5 \, k\Omega$, $L = 50 \, \mu H$, and $C = 200 \, pF$. Determine the resonant frequency, quality factor, and bandwidth.

P6.59. A parallel resonant circuit has $f_0 = 50 \, MHz$, $B = 2 \, MHz$, and $R = 1 \, k\Omega$. Determine the values of $L$ and $C$.

P6.60. A parallel resonant circuit has $f_0 = 10 \, MHz$ and $B = 200 \, kHz$. The maximum value of $|Z_p|$ is $10 \, k\Omega$. Determine the values of $R$, $L$, and $C$.

Section 6.8: Ideal and Second-Order Filters

P6.61. Name four types of ideal filters and sketch their transfer functions.

P6.62. An ideal lowpass filter has a cutoff frequency of $10 \, kHz$ and a gain magnitude of two in the passband. Sketch the transfer-function magnitude to scale versus frequency. Repeat for an ideal highpass filter.

P6.63. An ideal bandpass filter has cutoff frequencies of $9$ and $11 \, kHz$ and a gain magnitude of two in the passband. Sketch the transfer-function magnitude to scale versus frequency. Repeat for an ideal band-reject filter.

P6.64. Each AM radio signal has components ranging from $10 \, kHz$ below its carrier frequency to $10 \, kHz$ above its carrier frequency. Various radio stations in a given geographical region are assigned different carrier frequencies so that the frequency ranges of the signals do not overlap. Suppose that a certain AM radio transmitter has a carrier frequency of $980 \, kHz$. What type of filter should be used if we want the filter to pass the components from this transmitter and reject the components of all other transmitters? What are the best values for the cutoff frequencies?

P6.65. In an electrocardiograph, the heart signals contain components with frequencies ranging from dc to $100 \, Hz$. During exercise on a treadmill, the signal obtained from the electrodes also contains noise generated by muscle contractions. Most of the noise components have frequencies exceeding $100 \, Hz$. What type of filter should be used to reduce the noise? What cutoff frequency is appropriate?

P6.66. Draw the circuit diagram of a second-order low-pass filter. Suppose that $R = 1 \, k\Omega$, $Q_s = 1$, and $f_0 = 100 \, kHz$. Determine the values of $L$ and $C$.

*P6.67. Repeat Problem P6.66 for a highpass filter.

P6.68. A sinewave interference has been inadvertently added to an audio signal that has frequency components ranging from $20 \, Hz$ to $15 \, kHz$. The frequency of the interference slowly varies in frequency in the range $950$ to $1050 \, Hz$. A filter that attenuates the interference by at least $20$ dB and passes most of the audio components is desired. What type of filter is needed? Sketch the magnitude Bode plot of a suitable filter, labeling its specifications.

Section 6.9: Digital Signal Processing

P6.69. a. Develop a digital filter that mimics the action of the $RL$ filter shown in Figure P6.69. Determine expressions for the coefficients in terms of the time constant and sampling interval $T$. (Hint: If your circuit equation contains an integral, differentiate with respect to time to
obtain a pure differential equation.) b. Given $R = 10 \Omega$ and $L = 200$ mH, sketch the step response of the circuit to scale. c. Using a computer program of your choice, determine and plot the step response of the digital filter for several time constants. Use the time constant of part (b) and $f_s = 500$ Hz. Compare the results of parts (b) and (c).

*P6.71. Consider the second-order bandpass filter shown in Figure P6.71. a. Derive expressions for $L$ and $C$ in terms of the resonant frequency $\omega_0$ and quality factor $Q_s$. b. Write the KVL equation for the circuit and use it to develop a digital filter that mimics the action of the $RLC$ filter. Use the results of part (a) to write the coefficients in terms of the resonant frequency $\omega_0$, circuit quality factor $Q_s$, and sampling interval $T$. (Hint: The circuit equation contains an integral, so differentiate with respect to time to obtain a pure differential equation.)

**P6.70.** Repeat Problem P6.69 for the filter shown in Figure P6.70.