Homework 2

Reading: Book of Proof Sections 1.5-1.10, 2.1-2.3

Problems: Due Friday January 17, 2020 10:00pm PST in Canvas.
Use other sheets. There is not enough room for correct answers here. Show work.
($\emptyset$ is the empty set.)

1. (6 points) Give the set represented by each of the expressions below where
   \[ A_1 = \{-13, 1, 4, 87, \pi\}, \]
   \[ A_2 = \{-6, 4, 87, \triangle, \pi\}, \]
   \[ A_3 = \{-13, 87, \square, \pi\}, \]
   \[ A_4 = \{-6, 87, \triangle, \square, \pi\}. \]
   List each element in the set only once (i.e. \{1, 2\} instead of \{1, 2, 2\}).
   
   (a) \( A_1 \cup A_2 = \)
   (b) \( A_3 \cap A_4 = \)
   (c) \( A_4 - A_1 = \)
   (d) \( A_1 - A_4 = \)
   
   (e) \[ \bigcup_{i=1}^{4} A_i = \]
   (f) \[ \bigcap_{i=1}^{4} A_i = \]

2. (6 points) For each of the sets below fill in the corresponding regions of a general Venn diagram for
   3 sets. (The Venn diagram should have 3 sets in each case.)
   
   (a) \( \bar{A} \cup \bar{B} \cup C \)
   (b) \( A \cap \bar{B} \cap \bar{C} \)
   (c) \( \bar{A} - (B \cup C) \)

3. (4 points) In this problem the following notation is used for the set of integers and three of its subsets:
   
   The set of integers: \( \mathbb{Z} \)
   The set of shy integers: \( S \)
   The set of pale integers: \( P \)
   The set of worried integers: \( W \)
   
   (It is not necessary to understand what is meant by a shy, pale or worried integer.)
   
   Using only the symbols
   \(-3, \mathbb{Z}, S, P, W, \emptyset, \subseteq, \in, \cup, \cap, -, =, \{\}, (), \neq,\)
   express the following statements.
   
   (a) \(-3\) is pale but not worried.
   (b) No integer is shy or pale.
   (c) All worried integers are shy and pale.
   (d) Not all worried integers are shy.
4. (3 points) Suppose the three sets $A$, $B$ and $C$ satisfy the following:

\[
A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}
\]
\[
A - B = \{1, 2, 10\}
\]
\[
A - C = \{1, 2, 5\}
\]
\[
B - A = \{3, 4, 9\}
\]
\[
B - C = \{3, 4, 5\}
\]
\[
C - A = \{6, 7, 9\}
\]
\[
C - B = \{6, 7, 10\}
\]

What are $A$, $B$ and $C$?

5. (4 points) Let $P$ and $Q$ be the statements

\[
\begin{align*}
P & : x < 5 \\
Q & : x \text{ is an odd integer}
\end{align*}
\]

Rewrite each of the statements below using $P$ and $Q$ and logical connectives ($\sim$, $\land$, $\lor$, $\rightarrow$).

(Some of these statements might not be True and that’s okay.)

(a) $x \geq 5$
(b) $x$ is an odd integer or $x \geq 5$
(c) $x$ is an odd integer, but $x \geq 5$
(d) Whenever $x < 5$, $x$ is not an odd integer.

6. (4 points) Let $P$, $Q$ and $R$ be the statements

\[
\begin{align*}
P & : \text{Cats are furry.} \\
Q & : \text{Dogs are loud.} \\
R & : \text{I can have a pet.}
\end{align*}
\]

Express each of the statements below as an English sentence.

(a) $\sim R$
(b) $P \lor Q$
(c) $\sim P \rightarrow R$
(d) $P \lor Q \rightarrow \sim R$

7. (5 points) Use a truth table to determine the values of each of the logical expressions below. Both of your truth tables should have at least 5 columns (including the columns for the variables).

(a) $\sim (P \land \sim Q) \lor Q$
(b) $(P \lor Q) \land (P \lor R) \land \sim (Q \land R)$

8. (4 points) Convert each of the following statements into the form “If … then …” without changing their meanings. (Some of these statements might not be True and that’s okay.)

(a) Whenever a natural number is even, it must be at least 2.
(b) For a number to be prime, it must be greater than 1.
(c) A number is prime only if it is not even.
(d) Either $|n| > a$ or $|m| > a$ when $mn > a^2$.

9. (5 points) As discussed in class, given a finite set $S$ of size $n$ and an ordering $s_1, s_2, \ldots, s_n$ of the $n$ elements in $S$, we can represent the subsets of $S$ using bit vectors of length $n$ ($\{0, 1\}^n$). For a subset $A \subseteq S$, the corresponding bit vector $b(A) = (b_1, b_2, \ldots, b_n)$ where $b_i = 1$ if $s_i \in A$ and $b_i = 0$ if $s_i \notin A$.

Let $S$ be the elements from the four sets in Problem 1 ordered as $-13, -6, 1, 4, 87, \square, \triangle, \pi$.

(a) Give the bit vector corresponding to the subset $\{1, 87, \square, \pi\}$.
(b) Give the bit vector corresponding to the subset $\{-13, 1, 87, \triangle\}$.
(c) Give the bit vector corresponding to the subset $\{1, 4, -6, \square\}$.
(d) Give the bit vector corresponding to the subset $\emptyset$.
(e) Give the bit vector corresponding to $S$. 

2