Homework 10

Reading: Handouts 2 and 3 (in Canvas under Files) and BoP Sections 3.8 and 3.9

Problems: Due Friday March 13, 2020 10pm PDT in Canvas.

For the problems where you need to find the probability of an event (that’s problems 3 through 6) give

- the Experiment (if it’s the same as a previous problem say that)
- the Sample space,
- the Event space

and then calculate the probabilities. Simplify your answers as much as possible until you obtain a reduced fraction rather than a decimal number (for example, give the answer as $\frac{21}{89}$ rather than $0.236$).

1. (8 points) In this problem you will calculate the number of UCSC ID numbers that meet certain criteria or are needed to ensure certain criteria. UCSC ID numbers all have 7 digits from 0 to 9. We will assume that all digits can be 0 through 9. Be careful! There are subtleties lurking here.

   (a) (2 points) How many UCSC ID numbers have digits that sum to 8?
      (For example, 0200006, 1010033, 1020131 and 8000000 have digits that sum to 8.)
   (b) (2 points) How many UCSC ID numbers have digits that sum to 17?
   (c) (2 points) How many UCSC ID numbers must you have to guarantee that at least two of them sum to the same number?
   (d) (2 points) How many UCSC ID numbers must you have to guarantee that at least four of them have the same last 2 digits?
      (For example, 0189234 and 9033534 have the same last 2 digits, and 4089934 and 9033934 have the same last 2 digits.)

2. (6 points) These problems involve a fruit store that sells apples, bananas, kiwis, lemons, mangoes and oranges.

   (a) (2 points) How many ways are there to select a bag of 11 fruits?
      (The order of selecting the fruits is not important.)
   (b) (2 points) How many ways are there to select a bag of 11 fruits if there are only 3 kiwis available?
      (The fruit store only has 3 kiwis left, but plenty of the other fruits.)
   (c) (2 points) How many ways are there to select a bag of 11 fruits if there are only 3 kiwis and 3 bananas available?
      (The fruit store only has 3 kiwis and 3 bananas left, but plenty of the other fruits.)

3. (6 points) Suppose you roll a six-sided fair die twice.

   (a) (2 points) What is the probability that one or both rolls are 6’s?
   (b) (2 points) What is the probability that one or both rolls are even numbers (2, 4 or 6’s)?
   (c) (2 points) What is the probability that the sum of the two rolls is less than 6 or at least one of the rolls is an even number?

4. (6 points) Suppose the six-sided die you used in the previous problem is not fair. It is biased so that rolling a 6 is three times more likely than any other roll.
(a) (2 points) What is the probability that one or both rolls are 6’s?

(b) (2 points) What is the probability that one or both rolls are even numbers (2, 4 or 6’s)?

(c) (2 points) What is the probability that the sum of the two rolls is less than 6 or at least one of the rolls is an even number?

5. (8 points) The following questions are about a game in which you draw randomly from a sack containing 4 red balls, 4 blue balls, 3 green balls and 3 yellow balls. The ball is not replaced after each draw, and each ball in the sack is equally likely to be selected.

(a) (2 points) If you draw two balls from the sack without replacing them, what is the probability of drawing two red balls?

(b) (3 points) Starting over with all 14 balls back in the sack, what is the probability of drawing two balls that are the same color?

(c) (3 points) Starting over with all 14 balls back in the sack, what is the probability of drawing at least two yellow balls if you draw 4 times (without replacement)?

6. (8 points) As in the last problem these questions are about a game in which you draw randomly from a sack containing 4 red balls, 4 blue balls, 3 green balls and 3 yellow balls. But this time, the balls will be replaced after each draw. Again, each ball in the sack is equally likely to be selected.

(a) (2 points) If you draw two balls from the sack replacing the first ball before the second is picked, what is the probability of drawing two red balls?

(b) (3 points) Starting over with all 14 balls back in the sack, what is the probability of drawing two balls that are the same color (with replacement)?

(c) (3 points) Starting over with all 14 balls back in the sack, what is the probability of drawing at least two yellow balls if you draw 4 times (with replacement)?