1. Assume you have $n$ experts. In each trial, the algorithm chooses an expert $\hat{i}$ based on some distribution (chosen by the algorithm and known to the adversary), the adversary picks a loss vector $\ell \in \{0,1\}^n$ for the $n$ experts, and finally the algorithm incurs the expected loss of the chosen expert: $E(\ell, \hat{i})$.

You are to prove upper and lower bounds for sequences of trials where there is one expert with no loss.

(a) Give an adversary strategy for showing that any algorithm can be forced to have expected loss at least

$$\sum_{i=2}^{n} \frac{1}{i}.$$ 

(b) Show that Follow the Leader (ties broken uniformly) with dropout permutations has expected loss at most twice the above sum.

- First show that the unit rule holds for this algorithm.
- Then give a recurrence on the number of experts with perturbed loss zero and use this recurrence to prove the bound of twice the sum.

2. We saw in Lecture 2 that the Hedge algorithm with learning rate $\sqrt{\frac{\ln n}{\hat{L}}}$ has regret $\sqrt{2\hat{L}\ln n + \ln n}$ on sequences on which the loss of the best in hindsight loss $L^*$ is at most $\hat{L}$. In practice, we may not know the horizon $T$ or a good upper bound $\hat{L}$ on the loss of the best $L^*$. However we want algorithms that have good guarantees for all values of $L^*$, simultaneously, i.e. that keeps on operating forever.

Consider the following idea, called the doubling trick, to accomplish this. Run Hedge tuned for increasing values of $\hat{L}$. That is abort as soon as the loss of the best expert in the current segment breaks the current value of $\hat{L}$ and go to the next higher value of $\hat{L}$ and now restart and tune $\eta$ based on that new value. Try $\hat{L} = 1, 2, 4, \ldots$.

(a) Prove that the overall accumulated regret of Hedge with the doubling trick is bounded above by

$$O(\sqrt{L^*\ln n + \ln n})$$

on any sequences on which the best loss on the entire sequence is $L^*$.

(b) What about the tripling trick and friends? Prove a regret bound using other powers for $\hat{L}$. What is the best power to use?

3. I will set up a git repository where you can upload your Homework 1 report about the disk spindown problem. Your task is to evaluate three of your compatriots solutions. For each of the three solutions answer the following:

(a) What did you like/learn?

(b) What was unclear and what could be improved.

(c) Overall evaluation.

Per solution your evaluation should be about 1/2 of a page in length.