Shifting Experts on Easy Data

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Abstract

A number of online algorithms have been developed that have small additional loss (regret) compared to the best “shifting expert”. That is, there is a set of experts and the comparator is the best partition of the trial sequence into a small number of segments, where the expert of smallest loss is chosen in each segment. This work assumes worst-case data / loss sequences.

There has been a recent surge of interest in online algorithms that combine good worst-case guarantees with much improved performance on easy data. A practically relevant class of easy data is the case when the loss of each expert is iid with different means and the loss of the best and second best experts have a gap between their mean. In the full information setting, the FlipFlop algorithm by De Rooij et al. (2014) combines the best of the iid optimal Follow-The-Leader (FL) and the worst-case-safe Hedge algorithms, whereas in the bandit information case SAO by Bubeck and Slivkins (2012) competes with the iid optimal UCB and the worst-case-safe EXP3.

We ask the same question for the shifting expert problem. First, we ask what are the simple and efficient algorithms for instances of the shifting experts problem when the loss sequence in each segment is iid. And second, we ask how to efficiently unite the performance of such algorithms on easy data with worst-case robustness.

A particular intriguing open problem is the case when the comparator shifts within a small subset of experts from a large set under the assumption that the losses in each segment is iid.

Introduction

Much of the work on online learning algorithms has been done in the realm of learning well compared to a set of experts. In each trial, the learner (probabilistically) chooses an expert from the set, and at the end of the trial, the expert chosen by the learner as well as all other experts incur a loss. The goal is to find learning algorithms that have small regret which is defined as the total loss of the algorithm minus the loss of the best in a comparator class. In the base case, the comparator class is simply the set of experts and many algorithms have been developed for this case: The Weighted Majority and Hedge algorithms use weights that decay exponentially with the loss of each expert leading to multiplicative updates (Littlestone and Warmuth, 1994; Freund and Schapire, 1997), and the Follow the Perturbed Leader (FPL) algorithm perturbs the losses by additive noise and then applies FL (Kalai and Vempala, 2005; Hutter and Poland, 2005). All these algorithms are designed to achieve optimal regret when the losses of the experts are chosen by an adversary and the regret depends on the loss of the best comparator.

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The key to achieving small worst-case expected regret is that the algorithms make a probabilistic choice of the expert in each trial. Deterministic algorithms can be fooled with an alternating loss pattern. This focus on the worst-case lead to some elegant algorithms including an efficient minimax algorithm based on random walks (Abernethy and Warmuth, 2010). However the worst-case assumption might be overly pessimistic. If the losses are iid, then the simple deterministic FL algorithm is already minimax optimal (Kołodziej, 2013), and thus for most natural easy data, FL already works well and the fancy algorithms that guard against the worst-case are not needed.

**Shifting** The problem gets more challenging when the comparator can model data that changes with time. Now the comparator consists of a partition of the trials into segments and a choice of a best expert in each segment. Again the goal is to achieve small worst-case regret bounds with this richer class of comparators and an interesting set of new algorithm has been developed for this task (Herbster and Warmuth, 1998, 2001; Bousquet and Warmuth, 2002; Koolen et al., 2012), most notably multiplicative update plus a mixing update.

Most of the literature on shifting experts is with Vovk mixable loss functions (Vovk, 1998). Here the difference between iid and worst-case losses is perhaps insignificant, as the algorithms manage to balance the regret between all outcome sequences. However, in the linear/absolute/dot loss case this equalization property is lost, and a significant regret reduction can be achieved on some sequences. Worst-case bounds for the shifting experts problem with the dot loss have been obtained in Cesa-Bianchi et al. (2012).

**Problem: Efficient low regret algorithms and bounds for iid shifting experts**

We propose to focus on the shifting expert setting when the losses in each segment are iid with a gap, ie. the mean loss of the best and the mean loss of the next best expert differ by at least some fixed constant. There are many good simple algorithms for this setting and a first goal is to delineate between them in terms of their regret bounds. When a new section starts, then the algorithm must be able to recover the new best expert. Each of the below algorithm has a mechanism for doing that.

- FL on the best current partition (using dynamic programming for a cost of $O(nk)$ per trial). In the simplest case, $k$ is known to the algorithm.
- FL on a shifting window of length $w$ ending with the last trail. This requires $O(nw)$ storage between trials.
- FL on the total losses, where after each trial the total loss $L_i$ of each expert $i$ is reset to $\min(L_i, w + \min_j L_j)$, ie the total losses are kept at most $w$ apart. This requires $O(n)$ storage.
- FL on the total losses of each expert where the losses in the $q$-th last trial are multiplied by an exponentially decaying factor. This requires $n$ totals to be maintained.
- Use the weight updates designed for the worst case (ie a multiplicative update plus a mixing update, which typically requires $2n$ weights). However instead of drawing an expert from the current weight vector, the learner predicts with the expert of maximum weight.
The resulting regret bounds for these algorithms should $\log n$ per segment under the assumption that there is fixed gap between the mean loss of the best and the mean loss of the second best in each segment.

**Problem: Long-term memory for iid shifting experts**  One of the most interesting cases of expert algorithms is the long term memory setting where the best expert shifts within a small subset. In the worst case setting this was solved with a multiplicative update followed certain mixing updates (Bousquet and Warmuth, 2002; Koolen et al., 2012). What are the optimal algorithms for the long-term memory setting when the segments are assumed to be iid. Intuitively the algorithm has to locally follow the leader but help recovering to the new local leader when the (hidden) segment changes. It also has to recover faster towards experts that have done well in the past (long term memory).

If $n$ is size of the expert set and $k$ the size of the subset, then the question is whether there is an algorithm that achieves expected regret $\log n$ per member in the subset and $\log k$ per shift within the subset. Note that this is a version of Yoav Freund’s (Freund, 2000) original open problem which initiated long-term memory work of Bousquet and Warmuth (2002) that focused on the worst-case.

**Problem: Iid and worst-case-optimal shifting experts**  Ideally we want a single algorithm that can handle both the malignant worst-case and the benevolent iid case. For the base problem, this has been done in a whole line of research by fancy tunings of the Hedge algorithm (see Van Erven et al., 2011; De Rooij et al., 2014) and most recently with the FL algorithm applied to the dropout perturbed losses (Van Erven et al., 2014). In the bandit feedback model, the best of both worlds was achieved by Bubeck and Slivkins (2012). However it is not at all clear how these algorithm can be adapted to the shifting expert setting, let alone the long-term memory scenario.

**Wrap-up**  The importance of this open problem stems directly from the practical abundance of non-stationary data, and the desire to learn from it using online algorithms. We hope to have sketched an important family of problems. The details of the iid shifting experts problem can be worked out in many ways. One particular choice is whether we want to compare on each segment with the loss of the best expert on the sampled data or in expectation. Another is whether the segmentation is itself generated randomly, or chosen adversarially. In the simplest scenario, the algorithm knows the parameters of the problem like the gap, the number of trials, the number of segments in the comparator partition, the number of distinct best experts. The ultimate online algorithm should of course adapt to these practically unknown parameters on the fly.

**References**


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