Divide & Conquer

Dynamic Brute.
EM

Divide & Conquer:
Solve a # of smaller subproblems
Combine

Merge Sort

Split data set in two
Sort both recursively
Merge results

\[ M(n) = 2M\left(\frac{n}{2}\right) + cn \]

Recursion tree for computing

10 levels
O(n) work per level
O(n log n)
Dynamic proof.

- Keep table of subproblems
- Recurrence of current problem
  i.e. previously solve problems

Pierce used over and over again

\[ x_1 \ldots x_n \]
\[ y_1 \ldots y_m \]

LCS \( x, a, b, c, a, b \)

\[ y = b, c, b, b, a \]

\[ x, y \]

\[ T(i,j) \text{ longest common subsequence in } x_1 \ldots x_i, y_1 \ldots y_j \]

\[ \text{no match} \]
\[ T(i,j) = T(i-1,j-1) + 1 \text{ if } i,j > 0 \land x_i = y_j \]

\[ = \max(T(i,j-1), T(i-1,j)) \text{ if } i,j > 0 \land x_i \neq y_j \]

\[ = 0 \text{ if } i = 0 \text{ or } j = 0 \]

- What table
- Initialization
- Rec. & fill in
**UNAPPSACK**

Items $s_1, \ldots, s_n$

Does there exist a subset of size $k$?

$T(i, z) = \text{true if there exists a subset of } s_1, \ldots, s_i \text{ summing to } z$

\[ T(i, z) = \begin{cases} T(i, z-s_i) \lor T(i-1, z) & \text{if } i > 1, z \geq 1 \\ F & \text{if } i = 0, z > 0 \\ F & \text{if } z = 0, i > 1 \end{cases} \]