Lecture 16

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Our minimization problem

\[
\min_{w \in \mathbb{R}^n} \frac{1}{2} \|w - w_t\|^2 + w \cdot L \quad \text{s.t.} \quad \sum w_i = 1
\]

\[
U(w) = \frac{1}{4} \sum_i (w_i - w_{ti})^2 + \sum_i w_i L_i
\]

\[\nabla U(w) = \frac{1}{4} w - w_t + L\]

\[\nabla^2 U(w) = \frac{1}{4} I\]

↑ Hessian diagonal

Decoupled problems

Tight together by \( \sum w_i = 1 \) constraint
SLIGHTLY MORE GENERAL SETUP

\[ \text{min } f(x) = \frac{1}{2} x^T A x - c^T x \quad (x \in \mathbb{R}^n) \]

s.t.
\[ \ell \leq x \leq u \]
\[ a_i^T x = b_i \]

WHERE \[ A = \begin{pmatrix} a_i^T & 0 \\ 0 & d_u^T \end{pmatrix} \text{ HESSIAN} \]
\[ d_i > 0 \]

LEVEL CURVES OF \( f(x) \):

![Axis aligned ellipses]

PARTIAL LAGRANGIAN

\[ \phi(x, \lambda) = \frac{1}{2} x^T A x - c^T x - \lambda (a_i^T x - 1) \quad (\lambda \in \mathbb{R}) \]
\[ = \frac{1}{2} \sum_i d_i x_i^2 - c_i x - \lambda a_i x_i + \lambda \]

FOR FIXED \( \lambda \), UN CONSTRAINT SOL TO (x)

\[ d_i h_i - c_i - \lambda a_i = 0 \]
\[ h_i = \frac{(c_i + \lambda a_i)}{d_i} \]
CALL THIS SOL \( h(\lambda) \)
For fixed box constraint $\mathcal{C}$, the solution to (x) is

$$x(x) = \text{mid} \left( l, h(x), u \right)$$

SOL TO (xx) is SOL TO (x) s.t.

$$v(x) = a^T x(x) - \lambda = 0$$

$$v(x) = \sum a_i \text{mid} \left( l_i, \frac{\partial \text{mid}}{\partial l}, u_i \right) - \lambda$$

Non decreasing in $\lambda$
IDEA:

- FIND $a, b$ s.t. $v(a) \leq 0$
  - $v(b) \geq 0$

- SEARCH FOR $x^+$ s.t. $v(x^+) = 0$

- $x(x^+) IS$ SOLUTION TO ($x^+$)