BUILDING A GOOD CACHE BY COMBINING THE INITIAL SEGMENTS OF TWO LISTS

\[ |T_1| + |T_2| = c \]

ARCING :: HEURISTIC FOR SETTING A GOAL COMBINATION
- NO REFETCHING
  \begin{align*}
  & \text{ADJUST CACHE TOWARDS GOAL} \\
  & \text{SET CURRENT CACHE TO GOAL}
  \end{align*}

RELATIVE LOSS BOUNDS

\[ M_\text{alg}(S) \leq Q \cdot M^*(S) + \text{SMALL ADDITIONAL TERMS} \]

IDEALLY 1
IDEA: USE EXPERT FRAMEWORK
- ONE EXPERT PER COMBINATION

- HOW TO PREDICT?
  - RANDOM
  - DETERMINISTIC

1) REVIEW OF EXPERT BOUNDS
2) DISK SPINDOWN PROBLEM
3) CACHING W. 2 LISTS
4) " LISTS
PREDICTIVE MODEL

FOR $t=1, 2, \ldots$

GET PREDICTIONS OF EXPERTS

$(x_{t1}, x_{t2}, \ldots, x_{tn}) \in [0, 1]^n$

MAKE PREDICTION

$\hat{y}_t < \begin{cases} 0, 1 \end{cases}$ DETERMINISTIC

$\begin{cases} 0, 1 \end{cases}$ CONTINUOUS (PROBABILISTIC)

GET LABEL

$y_t \in \{0, 1\}$

INCR ERR LOSS

$|y_t - \hat{y}_t| \quad \text{ALG}$

$|y_t - x_{t,i}| \quad \text{EXPERT i}$

$w_{t+1,i} = \frac{w_{t,i} e^{-y_t L_{t,i}}}{z_t}$

$\hat{y}_t = \begin{cases} 1 \quad \text{if} \quad \bar{w}_t \cdot \bar{x}_t \geq \frac{1}{2} \\ 0 \quad \text{OTHERWISE} \end{cases}$ DET.

$\begin{cases} \hat{y}_t = \bar{w}_t \cdot \bar{x}_t \quad \text{CONTINUOUS} \\ = 1 \quad \text{WITH PROB.} \quad \bar{w}_t \cdot \bar{x}_t \\ = \bar{x}_{t,i} \quad \text{WITH PROB.} \quad w_{t,i} \end{cases}$ PROBABILISTIC

BOUNDS $w, \text{TUNED} \eta$

$L_{\text{ALG}} \leq \alpha L^* + O \left( \sqrt{L^* \ln n} + \ln n \right)$

$\alpha = 2 \text{ DET} \quad \alpha = 1 \text{ CONT. OR PROB.}$
DISC SPINDOWN:

\[ x_{t,i} = \frac{\text{TIMEOUT}}{c} \quad \text{IF} \quad \text{TIMEOUT} \in [0,1] \]

\[ \hat{y}_t = \overline{W_t} \cdot x_t \]

\[ y_t = \min \left( \frac{\text{SCALED IDLE TIME}}{1} \right) \quad \in [0,1] \]

JUSTIFICATION FOR MIN:

ALL LOSSES OF TIMEOUTS FLAT AFTER MAXIMUM TIMEOUT

\[ L_i(z) = L_i(\min(z,1)) \]

LOST NOT RELATED TO ABSOLUTE VALUE

\[ \text{Diagram:} \]

\[ \text{Diagram:} \]

\[ \text{Diagram:} \]
Use "decision theoretic setting".

For $t = 1, 2, \ldots$

Pick expert $i$ with prob. $w_{t,i}$

Get loss vector $L_t \in [0, 1]$.

Incur loss $L_{t,i}$ & expected loss $\bar{w}_t L_t$

$w_{t+1,i} = w_{t,i} e^{-\eta L_{t,i}} / Z_t$

Bound w. tuned $\eta$

$\sum_{t=1}^{T} w_t \bar{L}_t \leq (\min \sum_{i=1}^{C} L_{t,i}) + \sqrt{2 (\min \sum_{i=1}^{C} L_{t,i}) \ln n + \ln n}$

Does scaled loss have range $[0..1]$?

If $t_i \in [0..1]$ and $y_t = \min (\frac{\text{vote}(i, t))}{c}$

Scaled loss $s(y_t) \in [0..2]$

So bounds apply (last two terms in bound multiplied by 2).
ASSUMPTION THAT BEST TIMEOUT ∈ [0,1]

NOT WELL JUSTIFIED

\[ H_t := \frac{\sum_{i=1}^{t} \frac{1}{i}}{t} \quad \text{HARMONIC SUM} \]

\[ \propto \ln t \]

\[ H_0 = 0 \]

Set of \( t \) idle times

\[ \{ h_0, t, \ldots, h_{t-1}, t \} \quad \text{WHERE} \quad h_t = H_t - H_0 \]

Sum of tail

\[ h_{0,t} = H_t - H_0 = H_t \]

Largest

Total size of set = \( t \)

Total loss of timeout ≥ largest in set = \( t \)

Total loss of zero timeout = \( t \)

THM: Total loss of any timeout = \( t \)

THM: Best fixed timeout for any set of \( t \) times of total size \( t \)

IS \( \leq H_t \)

OPEN: Does there exist an \( ALG \) with bound of form

\[ L_{\text{ALG}}(S) \leq L^*(S) + O\left( L^*(S) \log t \right) + \log t \]

\( \uparrow \text{scaled} \)
BACK TO CACHING:

\[
\begin{align*}
\text{Losses } & \in [0, 1.3] \\
\text{Hit or Miss } & \checkmark
\end{align*}
\]

PROBLEM:
- How do we produce a cache from the weight vector over experts?

- Can't use weighted average
  \[
  \frac{\mathbf{w}_t}{\mathbf{x}_t}
  \]

  \[\uparrow\text{Binary, only available after request}\]
$n+1$ weights $w_0, \ldots, w_n$

**IDEA 1:** $\hat{y}_t = \frac{1}{n} \sum_{i=0}^{n} w_i \cdot i$ **MEAN**

- Why $i$

- When miss, need to half of total

weight by $\beta = e^{-\eta}$

$w_0 \quad w_1 \quad w_2 \quad w_3 \quad w_4 \quad \cdots \quad w_n$

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad \cdots \quad n$

**MEAN** $\cdot \alpha = \sum_{i=0}^{n} w_i \cdot i$

**LEFT** $\quad$ **RIGHT**

**TOTAL** $\quad$ **TOTAL**

**NOT NECESSARILY**

$50 \quad 50$

**SPLIT**
Can this be fixed? Different potential?

**Idea 2**: Predict w, weighted median weight

\[ w_0 \quad w_1 \quad w_2 \quad w_{i-1} \quad w_i \quad w_{i+1} \quad w_{n-1} \quad w_n \]

\[ \geq \frac{1}{2} \quad \frac{1}{2} \]

**Mean**

\[ \frac{1}{2} \quad \frac{1}{2} \]

Either mean upwards or downwards multiplied by \( \beta \) \[ \geq \frac{1}{2} \]

Bounds work even for det. case

\[ M_k \leq 2 M_k^0 + O(\sqrt{\log(1/n^2) + 1/n}) \]

Ideally mean does not move too fast?
RANDOMIZED ALG:
- \( i \sim w_i \)
- CONSTANT OF \( i \) IN FRONT OF \( M^k \)
- TOO MUCH REFETCHING

OPEN: IS CONSTANT OF \( i \) POSSIBLE
\( w \), NO REFETCHING, I.E.

\[ E(MA_L) \leq 1 \cdot M^k + O\left( \sqrt{M^k \ln n} + \ln n \right) \]

\( k \) LISTS \( D \)

\[ \Sigma \text{INITIAL SEMI}. = 1 \]

\( O(n^{k-1}) \) EXPERTS

- CAN WEIGHTED MEDIAN ALL
  BE IMPLEMENTED EFFICIENTLY

- EACH LIST PROVIDES MEDIAN
  \( \cdot \) TOGETHER = GLOBAL MEDIAN