1. Let $\Sigma$ be a set of functional dependencies. Let $(T, t)$ be a tableau query and let $(T_1, t_1), (T_2, t_2), \ldots, (T_p, t_p)$ be a sequence of tableau queries, where $T_1 = T$, $t_1 = t$ and $(T_{i+1}, t_{i+1}), 1 \leq i \leq p - 1$, is the result of chasing $(T_i, t_i)$ with a functional dependency in $\Sigma$. In addition, $(T_p, t_p)$ is a tableau query where no more functional dependencies in $\Sigma$ can be applied on $(T_p, t_p)$. Provide an algorithm to explain why $(T_p, t_p)$ can be obtained in polynomial time in the size of $(T, t)$ and $\Sigma$. Assume that there are $m$ functional dependencies in $\Sigma$ and $T$ is a tableau of arity $k$ (i.e., $k$ columns) with $n$ rows.

2. Suppose you are provided with the following two algorithms $A_1$ and $A_2$:

   Algorithm $A_1$ takes as input a conjunctive query $Q$ and a database instance $I$ and returns $Q(I)$.

   Algorithm $A_2$ takes as input two conjunctive queries $Q_1$ and $Q_2$ and returns $\text{TRUE}$ if and only if $Q_1 \subseteq Q_2$.

   Show the following:

   (a) Algorithm $A_1$ can be used to implement Algorithm $A_2$.
   
   (b) Algorithm $A_2$ can be used to implement Algorithm $A_1$.

3. Implement a query engine for conjunctive queries with your favorite programming language. The command line syntax for invoking the query evaluation engine should be similar to the following where we assume that the program is written in java.

   ```
   java cq-evaluator <queryfile> <dbfile>
   ```

   The above command returns the result of evaluating the conjunctive query specified in `<queryfile>` against the database instance specified in `<dbfile>`.

   Example of the contents of a `<queryfile>`:

   ```
   A(x, y) :- R(x, y, z), S(w, y, u).
   ```

   Example of the contents of a `<dbfile>`:

   ```
   R = \{ (1,2,3) (4,5,6) \},
   S = \{ (2,3,1) (1,3,3) (1,5,1) \},
   T = \{ \}, // no tuples in T
   U = \{ (1,2,8) (4,5,7) (1,1,1) (9,9,1) \}.
   ```
Executing the command

    java cq-evaluator <queryfile> <dbfile>

returns \( A(4,5) \).

To make your implementation easier, you may assume that the contents of \( \langle \text{queryfile} \rangle \) and \( \langle \text{dbfile} \rangle \) are always given in the correct format.

For convenience of grading, please make sure that your implementation can be executed on an SOE server.