1. Show that every rule-based conjunctive query is satisfiable. In other words, for every rule-based conjunctive query $Q$ that is specified over a relational database schema $R$, there is an instance $I$ of $R$ such that $Q(I)$ is non-empty.

2. Let $R$ be a binary relation scheme with attributes $A$ and $B$ and let $S$ be a unary relation scheme with attribute $C$. Prove or disprove that $\Pi_A(R \times S) = \Pi_A(R)$.

3. Let $Q_1$ and $Q_2$ be the following two queries:

   $Q_1(y, z) : \neg R(x, y), R(x, z)$

   $Q_2(x, y) : \neg R(x, y)$

   Show that $Q_1$ is not equivalent to $Q_2$. In other words, show that either $Q_1 \not\subseteq Q_2$ or $Q_2 \not\subseteq Q_1$.

4. (a) Define what it means for a query to be monotone.

   (b) Let $R_1$ and $R_2$ be two identical relation schemes and let $r_1$ and $r_2$ be the relations of $R_1$ and $R_2$ respectively. The difference of $r_1$ and $r_2$, denoted as $r_1 - r_2$, is defined to be the set of tuples that exists in $r_1$ but do not exist in $r_2$. Prove that the difference of $r_1$ and $r_2$ cannot be expressed with the SPCU algebra. (Hint: Show that every SPCU query is monotone but the query $r_1 - r_2$ is not a monotone query.)