Instructions: Answer all questions concisely.

1. Assume that \textit{dom} is a set of \( n \) elements and \( r \) is an instance of a binary relation schema \( R \) over \( \textit{dom} \times \textit{dom} \).
   - What is the largest possible size of \( r \)?
   - What is the largest possible result of the following query?
     - \( \text{Ans}(x, z) \leftarrow R(x, y), R(y, z) \).

2. Prove that the SPCU algebra is monotonic. That is, prove that the composition of SPCU operators is monotonic. (Hint: prove by induction.)

3. Let \( Q_{\text{min}} \) be a minimal conjunctive query. Show the following:
   - If \( Q \) is a conjunctive query and \( Q \equiv Q_{\text{min}} \), then every containment mapping \( h \) from \( Q \) to \( Q_{\text{min}} \) is such that for every subgoal \( S(x_1, \ldots, x_k) \) of \( Q_{\text{min}} \), \( S(h(y_1, \ldots, y_j)) = S(x_1, \ldots, x_k) \) for some subgoal \( S(y_1, \ldots, y_k) \) of \( Q \).

   (Hint: Prove by contradiction. Assume that there is a containment mapping \( h: Q \rightarrow Q_{\text{min}} \) such that for some subgoal \( S(x_1, \ldots, x_k) \) of \( Q_{\text{min}} \), none of the subgoals in \( Q \) maps to \( S(x_1, \ldots, x_k) \) under \( h \). Show how this will contradict the minimality of \( Q_{\text{min}} \).)