Games of Imperfect Information

1. Historical Background

- Pre’67: issue of imperfect information is important, but considered intractable, “incomplete”
- John Harsanyi showed a general approach, made it sort of incomplete information
- The general idea can be shown in the following example

2. Example 1, Trust Game, Imperfect Information

<table>
<thead>
<tr>
<th>#1</th>
<th>T</th>
<th>#2</th>
<th>C</th>
<th>(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(0,0) (-1, X)

- #2 player knows his own type (vicious or nice)
- #1 player knows that X can be either 2 (“vicious”) or 0 (“nice”)
- #1 player has imperfect information about player #2

The game can be formalized in extensive form by adding nature N

- P is prior belief of a nice and vicious people
- P can be updated dynamically according to Bayesian rule
Bayesian Nash Equilibrium Solution:

\[ Ef_1(T) = (1-p)*1 + p*(-1) = 1 - 2p \geq 0, \]

\[ Ef_1(N) = 0; \] So \( Ef_1(T) \geq Ef_1(N) \) iff \( P \leq 0.5 \).

Thus the BNE solution is \( (T, CD) \) if \( P \leq 0.5 \) and is \( (N, CD) \) if \( P \geq 0.5 \).

Harsanyi Algorithm can be formalized as:
1. Encapsulate the incomplete information into a set of possible “types”
2. Specify type-contingent games
3. Tie these games together with an initial Nature move including priors
4. solve for the NE etc. in larger game called BNE of basic game.

3. Signaling Games
There are 3 basic types of games of incomplete information: Principal/Agent, Screening and signaling. Here I’ll just discuss the last type in simplest form.

3.1 Biological Example: Monarch and Bird

Monarch butterflies eat milkweed to make themselves taste bitter for bird, the monarch also grow conspicuous markings to signal it doesn’t taste good for the bird. Slowly the bird associates monarch butterfly mark as a signal for bad taste and thus spares the monarch butterflies. Monarch butterflies prosper via successful signaling.

3.2 Social Science Example: Education as a signal for talent

Example A: Pooling equilibrium

\[
\begin{array}{c|c|c|c}
#1/H & #1/L & #2_N & #2_E \\
1/3 & 2/3 & & \\
\hline
E & M & C & N \\
10,10 & 4,4 & 10,0 & 4,4 \\
\hline
M & N & E & M \\
6,10 & 0,4 & 6,10 & 0,4 \\
\end{array}
\]

E: education, cost of 4 born by worker
N: no education
H: high productivity (1/3 of #1 population)
L: low productivity (2/3 of #1 population)
M: managerial job
C: routine clerk job

Solution: pooling equilibrium

If #1/H choose E, get M(6), #1/L choose N, get C (4), then after Bayesian Updates, #1/L will choose E as well since (6>4). Then BNE is a pooling equilibrium. Education as a signal can’t separate H/L workers.

Example B: Separating Equilibrium

Here the education cost will be different for H/L workers, it will cost 7 for L and 4 for H. #1/L will not choose E, even if he knows that education will bring him a managerial job, but the payoff is only 3, which is less than 4 if he chooses no-education and get a clerk job. Only #1/H will go for the education (6>4). So the education as a signal effectively separates H/L workers.

3.3 Economics Example: The market for “lemons”

George Akerlof (2001 Nobel Laureate) applied asymmetric information game to the used car market. The illiquidity of the used car market can be explained by the imperfect information about the quality of used cars. The seller knows his car is peach or lemon while the buyer doesn’t know the quality. The buyer usually offers an average price between lemon and peach since he doesn’t know the quality, the seller with peach won’t sell the car since the price is below its real value, thus only the lemons will be on the
market, the buyer knows that and decide to only to pay the price for the lemon, peach won’t be traded due to the imperfect information.

3.4 Formal definition of Separating/Pooling Equilibrium

Consider the following 2-player game. Sender(player#1) has a true type \( \theta \in \Theta \) known to himself but not to the other player. Nature selects type according to distribution \( \mu(\theta) \). Sender moves first and choose signal (“message”) \( m \in M \). Receiver (player #2) observes \( m \), then chooses action \( a \in A \). The payoff functions are \( f_1(m,a,\theta) \) and \( f_2(m,a,\theta) \). For example, in the education/employment game mentioned earlier, \( \Theta = \{H,L\} \), \( M = \{E,D\} \) and \( A = \{M,C\} \) and the payoffs were given at the terminal nodes.

The general definition of an equilibrium (BNE) for a signaling game is as follows:

BNE is a strategy profile \((m^*(\theta), a^*(m), \mu(\theta|m))\) (possibly mixed)

s.t.  
(1) \( m^*(\theta) \in \text{argmax} \ f_2(m,a^*(m), \theta) \) for all \( \theta \) (#1)
(2) \( a^*(m) \in \text{argmax} \ E\{f_1(m,a, \theta) \mid m \} \) for all \( m \) (#2)
(3) \( \mu(\theta \mid m) \) is derived from prior \( \mu_0 \) & \( m^*(\theta) \) via Bayes Theorem

Two important types of BNEs are:
Pooling equilibrium: \( m^*(\theta) \) is a constant, e.g., \( M_h^* = M_l^* \) in education/emp. game
Separating equilibrium: \( m^*(\theta) \) is 1:1, e.g., \( M_h^* = E \neq N = M_l^* \),