CMPS272: Game Theory

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Hawk Dove Game, Continued

In normal form, the payoff matrix for Player 1 is:

\[
\begin{array}{c|cc}
 & H & D \\
\hline
H & \frac{v-c}{2} & V \\
D & 0 & \frac{V}{2}
\end{array}
\]

where $V$ is the resource and $C$ is the cost variable, making the payoff $\frac{v-c}{2}$ when two Hawks meet. $H$ is the frequency of hawks, and $D$ is the frequency of doves. Note that

\[
D = 1 - H.
\] (1)

Furthermore,

\[
W_H = H \left( \frac{v-c}{2} \right) + DV
\] (2)

\[
W_D = 0 + D \frac{V}{2}
\] (3)

\[
\bar{W} = DW_D + hW_H
\] (4)

Suppose we wish to find the equilibrium. We could attempt to find the difference $H(t) - H(t+1)$ as $t$ (time) tends to infinity. This can be very difficult. Alternatively, we solve instead for the value $\bar{H}$ that makes

\[
W_H = W_D.
\] (5)

That is

\[
W_H = W_D
\]
from (2) and (3) this is

\[ H \left( \frac{v - c}{2} \right) + DV = D \frac{V}{2} \]

substituting in using (1) gives

\[ H \left( \frac{v - c}{2} \right) + (1 - H) V = (1 - H) \frac{V}{2} \]

\[ (1 - \hat{H}) V = V - \hat{H}(C + V) \]

and we solve for \( \hat{H} \). Of course, this only works if you start out with parameters that make sense \((V > 0)\).

This operation is what is referred to as selection dynamics. Your Excel simulations should converge to \( \hat{H} \). Oscillations arise because we are dealing with discrete time simulations. Also, it would be interesting to know what it takes to ensure that \( W_H > 0 \).

3 Kinds of 2-Player Games

1. **Hawk-Dove**, as described above, and in the last lecture.

2. **Coordination Game** (including Cheaptalk and Secret Handshakes):

   These Games are defined by the following payoff matrix

   \[
   \begin{array}{c|c}
   a_1 & 0 \\
   \hline
   0 & a_2 \\
   \end{array}
   \]

   where \( a_1 > a_2 > 0 \). There are several Nash Equilibria \(((a_1, a_1), (a_2, a_2))\), and the unstable \((0, 0)\), but the best response is \((a_1, a_1)\). This “best response” in this case is referred to as the response of the highest social efficiency.

   \[
   \text{Indw:} \quad \frac{P_1 + P_2}{2}
   \]

   In a game such as this if you play with the consensus or you’re “screwed”.

Another example: (Cheaptalk, or secret handshake). Same payoff matrix as above, but we have that the population always plays strategy \( a_2 \). “Mutants” enter the game which have the following “powers”:

- the can recognize other mutants
and if it recognizes another mutant it plays $a_1$, the better strategy
otherwise, it goes with the status-quo, playing $a_2$ in order to assure
positive payoff.

A “mutant” is a key player in most social dynamics. When they recognize
other mutants $a_1$ is played because its “cost-less”. In this sense, mutants
have a secret handshake with other “mutants” which allow greater payoff
under certain circumstances.

3. Prisoner’s Dilemma

Example: A payoff matrix from the perspective of Player 1. A district
attorney gains leverage over suspects (Player’s 1 and 2) offering confession
in exchange for a shorter sentence. Payoffs are in terms of number of years
in prison (negated— clearly less years are desired).

<table>
<thead>
<tr>
<th></th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>-2</td>
<td>-5</td>
</tr>
<tr>
<td>Defect</td>
<td>-1</td>
<td>-3</td>
</tr>
</tbody>
</table>

Defecting is the best strategy, but cooperating gives the best result (in
the expectation). This is the socially efficient result.

2 Kinds of 3-Player games

1. Hawk-Dove-Bourgeois

Key Idea: Bourgeois (B) is the average of strategies H and D. When
a bourgeois encounters a “territory owner” then it acts as a Dove (D).
Otherwise, when it is itself the territory owner it acts as a Hawk (H). In
this way Bourgeois have an advantage over the H strategy. The payoff
matrix goes as follows:

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>D</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>$\frac{v}{2}$</td>
<td>$\frac{v}{2}$</td>
<td>$\frac{v}{2}$</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>$\frac{v}{2}$</td>
<td>$\frac{v}{2}$</td>
</tr>
<tr>
<td>B</td>
<td>$\frac{v}{2}$</td>
<td>$\frac{v}{2}$</td>
<td>0</td>
</tr>
</tbody>
</table>

This game will need to be simulated in an Excel spreadsheet by this time
next week. In this spreadsheet it is important to explore the boundary
conditions!

2. Rock-Paper-Scissors

Or more correctly Rock-Scissors-Paper (R-S-P).
Rock-beats-Scissors-beats-Paper-beats-Rock

RSP has what is called *fitness transitivity*: one strategy beats another, and is itself beaten by the third. The payoff matrix for this game is as follows:

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>P</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>P</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>S</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

where different choices of $a$ result in different (sometimes interesting) behavior. Another (more interesting) form of the game is:

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>P</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>1</td>
<td>$2 + a$</td>
<td>0</td>
</tr>
<tr>
<td>P</td>
<td>0</td>
<td>1</td>
<td>$2 + a$</td>
</tr>
<tr>
<td>S</td>
<td>$2 + a$</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

These games can be visualized in diagrams like those below. When $a = 0$ we have

![Diagram](image1.png)

when $a < 0$ it looks like

![Diagram](image2.png)
and when $a > 0$ the picture becomes: