Name:

Exam 2, Fall 2000  
CMPS 262 Computer Animation

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1  **Euler Interpolation (10 pts)**

1. Assume, as in the homework, that you want to interpolate using Euler angles between two coordinate frames. The first frame is rotated 90 around the Y axis from the original worldspace orientation. The second frame is rotated 90 degrees about the X axis from the original worldspace orientation. What are the direction vectors for the X, Y, and Z axis of an interpolated coordinate frame that is 50% of the way between these two frames. (In other words, what is the matrix that represents this interpolation.) Assume rotations are applied in the order Rx*Ry*Rz*Vectors.

   **Answer** (45,45,0)
   
   .7 0 .7
   .5 .7 -.5
   -.5 .7 .5
   x y z axes
2 Quaternions (10 pts)

1. What is the quaternion for a rotation of -90 degrees around the Z axis, and the quaternion for a rotation of 90 degrees around the X axis?

2. What is the quaternion for a slerp interpolation that is halfway between these two quaternions?

**ANSWER**
-90 degrees around Z: \( \cos 45 = 0.707 \sin 45 = 0.707 \)
\((0.707, 0, 0, -0.707)\)

90 degrees around X: \( \cos 45 \cdot 0.707 \sin 0.707 \)
\((0.707, 0.707, 0, 0)\)

slerp half way = same as linear interpolation half way
\((0.707, 0.3535, 0, -0.3535)\)

normalized \((0.82, 0.4, 0, -0.4)\)
3 Hierarchies (15 pts)

A 3d arm consists of an upper arm, lower arm, and hand. The lengths are (3, 2, 1) respectively. The upper arm is the root of the hierarchy. The upper arm coordinate frame originates at the world coordinate frame, but its longitudinal (local z) axis is rotated by -90 degrees around the world x-axis. The lower arm is rotated by 45 degrees around its parent’s y-axis. The hand is rotated -45 degrees around its parent’s z-axis.

1. Show the sequences of matrices that take a point from the hand state space to the lower arm state. READ THAT CAREFULLY!

\[
\begin{align*}
\text{ANSWER} & \quad 1 & 0 & 0 & 0 & .7 & .7 & 0 & 0 \\
& 0 & 1 & 0 & 0 & * & -.7 & .7 & 0 & 0 \\
& 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 \\
& 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
\end{align*}
\]

2. Where is the point at (1,0,0) in the local hand space when converted to the lower arm coordinate frame. READ THAT CAREFULLY ALSO!

\[
\text{ANSWER (0.707, -0.707, 2)}
\]
4 Inverse Kinematics (10 points)

A 2D hierarchical object consists of a root segment of length 1 and another segment of length 1 that lies at the end of the first segment. The original orientation is shown below. The end effector is the end of the second segment.

1. What is the Jacobian matrix for this problem?
   ANSWER $(-1, -1) \over (1, 0)$ inverse is $(0,1)$ over $(1,1)$

2. Using the Jacobian, find how much to rotate each segment such that the end of the second segment is at $(2, 0)$.
   ANSWER $(-1, 0)$.  

5  Dynamics (15 pts)

5.1  Point Masses (10 pts)

You have two points, each of mass 0.5 kilograms at positions (10,15) and (20,15). Both are moving at velocity (10,0). There is a spring between them with rest length 11 and spring constant 0.5. Use time steps of 1 second, and the Euler method of integration. There is no gravity.

a. What is the spring force applied on the right mass at time 0.

**ANSWER**
a = F/m = 0.5/0.5 = 1.0

b. What is the position of the right mass at time 1?

**ANSWER**
p = 20 + 10 * 1 + 0.5 * 1 * 1 = 30.5

5.2  Rigid Body Dynamics (5 pts)

If a force of (1,0,0) is applied to a rigid body at location (0,1,0) in its local coordinate system, what torque is produced? Show the equation you used to find this.

**ANSWER**
\[ \mathbf{r} \times \mathbf{F} = (0,1,0) \times (1,0,0) = (0,0,-1) \]
6  Collisions (10 pts)

6.1 Inside (5 points)

Prove using equations that the point (2,-1) is outside of a 2D square with sides of length 2 that is centered at the origin (0,0,0). (Hint: Use the minimum number of equations.)

**ANSWER**
check with right edge normal (1,0) point to point is (1,0)
dot product is positive to outside as square is convex.

6.2 Quadtrees (5 pts)

Put the following list of points into a quadtree in the order given using the method described in class for deciding whether a point can collide with a bounding box: A=(1,-3), B=(2,4), C=(3,-4), D=(3,5). Label the edges of the quadtree with ll (lower left), lr (lower right), ul (upper left), and ur (upper right).

**ANSWER**
A ur B ur D
    lr C
7 Constraints (10 pts)

Here is a two-dimensional problem. There is a stick that starts at (0,0) and ends at (1,1). You want it to start at (1,0) and end at (3,2). You want to solve this using energy constraints, as described in the paper. You can translate the first point, and rotate and uniformly scale the whole stick.

a. Give the constraint equations in the form of an objective function. Define all variables. Be specific and use actual numbers where appropriate.

**ANSWER**

for example

\[(x - 1)^2 + (y)^2 + (S-2)^2\]

---

b. Give the equations that specify how the parameters change during the first iteration. If the equation includes a gradient, show the actual values, not just the symbol. Use a time step of 0.1

**ANSWER**

rate of change \((-2,0,-2)*0.1 = (-.2,0,-.2)\)

\[
x = x - (-0.2) = .2
\]

\[
y = y - 0 = 0
\]

\[
s = 1 - (-.2) = 1.2
\]
8  Constraint Methods (10 pts)

Know something about how constraint methods work, including a general understanding of the papers we discussed in class. Know the differences between ordinary constraints and spacetime constraints. Know the differences in the spacetime methods that we discussed. What are the issues involved in choosing a method?

9  Motion Transformation Methods (10 pts)

Know something about how motion transformation methods work. What are the issues in making it successful? What the differences in the approaches taken in papers that we have discussed.