STREAMLINE SETUP (NO LABELS)

FOR $t = 1$ TO $T$

CHOOSE AN EXPERT $i$

GET LOSS VECTOR $\tilde{L}_t \in [0,1]^N$

INCR LOSS $L_{t,i}$

GOAL: ACHIEVE SMALL REGRET

TOTAL LOSS OF ALG - TOTAL LOSS OF BEST

ALG $T$: FOLLOW THE LEADER

- ALWAYS CHOOSE THE BEST EXPERT
  (BRAKE TIES ARBITRARILY)

ADVERSARY:

- CHOSEN EXPERT 1 UNIT OF LOSS
- ALL OTHERS LOSS 0

FACTOR OF $n$ OFF

LOSS OF ALG $T$

LOSS OF BEST $\left\lfloor \frac{T}{n} \right\rfloor$
**ALG II: RANDOMIZED WEIGHTED MAJORITY**

**ALGORITHM**

**PROBABILISTIC CHOICE OF EXPERT**

\( \overline{w}_t \): probability vector used at trial \( t \)

\( w_{t,i} \): "believe" at trial \( t \) that \( i \) is best

\( \overline{w}_t = \left( \frac{1}{m}, \ldots, \frac{1}{m} \right) \)

**FOR** \( t = 1 \) **TO** \( T \) **DO**

**CHOOSE EXPERT** \( i \) **WITH** \( \text{PROB.} \ w_{t,i} \)

**GET LOSS VECTOR** \( \overline{L}_t \)

**INCREMENT LOSS** \( L_{t,i} \) **OR**

**EXPECTED LOSS** \( \overline{w}_t \cdot \overline{L}_t = \sum_{i} w_{t,i} L_{t,i} \)

\[ w_{t+1,i} = \frac{w_{t,i} e^{-\eta L_{t,i}}}{\sum_i w_{t,i} e^{-\eta L_{t,i}}} \]

**↑ NORMALIZATION**

\( \eta > 0 \) **LEARNING RATE**

\[ e^{-\eta} = \beta \]

\[ e^{-\infty} = 0 \]
\[ w_{t+1, i} = \frac{e^{-\eta L_{t, i}}}{Z_t} \]

As \( \eta \to \infty \), all weight placed on best & WMR becomes "follow the leader"

\[ w_{t+1, i} = \frac{w_{t, i} e^{-\eta L_{t, i}}}{Z_t} = \frac{w_{t, i} e^{-\eta L_{t, i}}}{\sum w_{t, i} e^{-\eta L_{t, i}}} \]

\( \eta = 0 \): weights unchanged

\( \eta > 0 \): gradually move weight to experts with low loss

"Soft Min"

\( \eta < 0 \) \rightarrow high loss
ANALYSIS:

**POTENTIAL:**

\[ P_t = - \ln \sum_i w_{i, i} e^{-\eta L_{t, i}} \]

↑

**DUE TO NORMALIZATION**

\[ P_{t+1} - P_t = - \ln \sum_i w_{i, i} e^{-\eta L_{t+1, i}} + \ln \sum_i w_{i, i} e^{-\eta L_{t, i}} \]

\[ = - \ln \frac{\sum_i w_{i, i} e^{-\eta L_{t+1, i}} e^{-\eta L_{t, i}}}{\sum_i w_{i, i} e^{-\eta L_{t+1, i}}} \]

\[ = - \ln \sum_i w_{t, i} e^{-\eta L_{t, i}} \]

\[ \geq - \ln \sum_i w_{t, i} (1 - (1 - e^{-\eta}) L_{t, i}) \]

\[ e^{\eta x} \leq 1 - (1 - e^{-\eta}) x \quad \text{for } x \in [0, 1] \]

\[ \ln(1-x) \leq -x \]

\[ \ln(1-x) x \leq (1 - e^{-\eta}) \bar{w}_t \cdot \bar{L}_t \]

**DROP OF POTENTIAL**

\[ \geq (1 - e^{-\eta}) \text{ LOSS OF ALG.} \]
\[ \sum_{t=1}^{T} P_{t+1} - P_t \geq (1-e^{-\eta}) \sum_{t=1}^{T} w_t \cdot L_t \]

Lower Bound

\[ \sum_{t=1}^{T} P_{t+1} - P_t = P_{T+1} - P_1 = 0 \]

\[ = -\ln \frac{1}{\bar{w}_{t,i}} e^{-\eta} L_{t,i} \]

\[ \leq -\ln \bar{w}_{t,i} e^{-\eta} L_{t,i} \]

\[ = -\ln \bar{w}_{t,i} + \eta L_{t,i} \]

Upper Bound

\[ \sum_{t=1}^{T} w_t \cdot L_t \leq \frac{\ln \frac{1}{\bar{w}_{t,i}} + \eta L_{t,i}}{1-e^{-\eta}} \]

If \( \bar{w}_i = \left( \frac{1}{n} \cdots \frac{1}{n} \right) \) THEN \( \ln \frac{1}{\bar{w}_{t,i}} = \ln n \)

- Can handle lots of experts

\( \eta = 1 \) gives bounds of the form

\[ L_{ALC} \leq a \text{ loss of best} + b \ln n \]

\( a, b \geq 1 \)

If \( \eta \) tuned as function of \( n \) & \( \hat{L} \) then

\[ \sum_{t=1}^{T} w_t L_t \leq \frac{\text{ms} L_{t,i} + \sqrt{2\hat{L} \ln n}}{\hat{L}^*} + \ln n \]

If \( L^* \leq \hat{L} \)
BIG PICTURE
- WE USED EXPONENTIAL WEIGHTS
  AND SOFTMIN TO ACHIEVE REGRET BOUNDS

- EXPECTED LOSS BOUNDS HOLD FOR
  ARBITRARY SEQUENCES

- EXPECTATION WRT INTERNAL RANDOMIZATION
  OF ALG

- LOGARITHMIC DEPENDANCE ON # OF EXPERTS
  , TYPICAL FOR "MULTIPlicative"
  UPDATES

QUESTIONS:
- LOWER BOUNDS ?
- MOTIVATION OF UPDATES ?
- WHERE DID THE POTENTIAL
  COME FROM ?
- WHAT ABOUT OTHER LOSS FUNCTIONS ?
- COMPARE AGAINST BEST LINEAR COMBINATION
  OF EXPERTS ?
Lots of "stupid" experts are "specialized" combined to something better.

Later: Boosting

- Iteratively builds
  small linear combination
  of weak hypothesis

For fun: Bug machine

Many stupid bugs better
than one smart bug

- Variety is asset in changing environment