1. Compute the dual of the following minimization problem, (where the \( \mathbf{a}_i \) are vectors in \( \mathbb{R}^N \)):

\[
\begin{align*}
\min_{\alpha \in \mathbb{R}, \mathbf{d} \in \mathbb{R}^N_{\geq 0}} & \quad \alpha + \eta \sum_{n=1}^{N} (d_n \ln d_n - d_n) \\
\text{s.t.} & \quad \mathbf{a}_i \cdot \mathbf{d} \leq \alpha, \quad \text{for } 1 \leq i \leq k
\end{align*}
\]

Hints: Form a Lagrangian, take derivatives and solve for \( d_n \). Plug this solution into the Lagrangian. Clearly state the dual optimization problem.

2. Consider the Unnormalized Hedge update:

\[
\mathbf{w}_{t,i} := \mathbf{w}_{t-1,i} e^{-\eta L_{t,i}}.
\]

(a) Show that this update is the solution to the following optimization problem:

\[
\mathbf{w}_t = \arg \min_{\mathbf{w}} \sum_{i=1}^{n} \left( w_i \ln \frac{w_i}{w_{0,i}} + w_{0,i} - w_i \right) + \eta \mathbf{w} \cdot \mathbf{L}_{\leq t}.
\]

The value of this optimization problem serves as a potential.

(b) Compute the potential \( P_t \) by plugging the update \( \mathbf{w}_t \) into the objective of the minimization problem.

(c) Show the following lower bound on the drop of the potential

\[
P_t - P_{t-1} \geq (1 - e^{-\eta}) \mathbf{w}_{t-1} \cdot \mathbf{L}_t.
\]

3. Consider the following transfer function:

\[
h(a) = \frac{\exp(a) - \exp(-a)}{\exp(a) + \exp(-a)}.
\]

(a) What is \( h^{-1}(y) \)?

(b) What is the pre and post matching loss?

Try to simplify your answers as much as possible.

The post loss should be some sort of relative entropy.

(c) How is this all related to logistic regression?