Problems to be turned in:

1. (5 pts) Exercise 34.2-1 on page 1065 (Show GRAPH-ISOMORPHISM is in NP, see page 1171 for the definition of when graphs are isomorphic).

2. (5 pts) Assume that we have an a polynomial time algorithm for deciding whether or not a string is in the CIRCUIT-SAT language (see page 1072). Use this algorithm as a subroutine in a polynomial time algorithm that, when given the description of a satisfiable circuit, finds an assignment of boolean values to the inputs that causes the circuit to output the bit 1.

3. (10 pts) Consider the following MANY-DISJOINT-SETS decision problem.

   Given an integer \( n \), a set \( S \) of subsets of \( \{1, 2, \ldots, n\} \), and a bound \( k \), does \( S \) contain \( k \) mutually disjoint subsets?

   For example, if given the input: \( n = 4, S = \{\{1, 2, 3\}, \{1, 4\}, \{3, 4\}, \{2, 3\}\} \), and \( k = 2 \), then the answer is “yes” since there two subsets in \( S \) (\( \{1, 4\} \) and \( \{2, 3\} \)) that are disjoint. However, no three of the subsets in \( S \) are disjoint.

   Show that the language consisting of strings encoding “yes” instances of the MANY-DISJOINT-SETS problem is NP-complete. (Hint: there is a reduction from CLIQUE that is not too difficult).

Seven problems not to be turned in:

1. (22.2-6 in the text) Give an example of a (connected) directed graph \( G = (V, E) \), a source vertex \( s \in V \), and a set of tree edges \( E_\pi \) such that:

   1. for each \( v \in V \), the (simple) path from \( s \) to \( v \) using only tree edges in \( E_\pi \) is a shortest path from \( s \) to \( v \) in \( G \), and
   2. the edges \( E_\pi \) are never found by a BFS on \( G \) no matter how the adjacency lists of \( G \) are ordered.

2. (22.3-8 in the text) Give a counterexample to the following conjecture about a depth-first searches in an arbitrary directed graph \( G \).

   If \( G \) contains a path from \( u \) to \( v \) and \( u.d < v.d \) in the depth-first search, then \( v \) is a descendant of \( u \) in the depth-first forest produced by the DFS.
3. (34.1-6 on page 1061) Prove the class P, viewed as a set of languages, is closed under union, intersection, concatenation, complement, and Kleene star.

4. 22.2-1 on page 601 (BFS example)

5. 22.3-1 on page 610 (DFS edge types)

6. 22.3-2 on page 610 (DFS example)

7. Show that polynomial time reductions are transitive, i.e. if $L_1$, $L_2$, and $L_3$ are languages such that $L_1 \leq_p L_2$ and $L_2 \leq_p L_3$, then $L_1 \leq_p L_3$. (This is a rephrasing of exercise 34.3-2).