Due Thursday April 8, 20 pts

This homework is to be done in groups of 3 (or 2 if necessary), students may select their own groups. When I say/write “show” it means prove, and when I say/write “prove” it means ”prove formally”.

There are “Problems not to be turned in” on the second page, and reading assignments will be updated on the class web page.

3 Problems to be turned in

1. (5 pts) Prove formally that for any fixed (i.e. constant) $b > 1$, the function $\log_b n$ is in $\Theta(\log_2 n)$. Use the c and $n_0$ style definition from the book.

2. (5 pts) Find a pair of increasing functions $f(n)$ and $g(n)$ (both from the positive integers to the positive integers) such that $f(g(n))$ is not in $O(g(f(n)))$. Prove that $f(g(n))$ is not in $O(g(f(n)))$.

3. (10 pts) Prove formally by induction that:

   Any undirected graph having the same number of edges as nodes contains a cycle.

   (Hint: you may use without proof the fact that any graph on $n$ vertices with $n$ edges has at least one vertex of degree $\leq 2$, and consider removing such a vertex in the inductive step.)
Other Problems (not to be turned in)

1. Problem 3-3 part a) on page 61-62 (or page 58 of 2nd edition) (ranking functions); (note: there are a lot of functions, ranking many of them is OK.)

2. Prove by induction that if $T$ is a non-empty full binary tree with $N(T)$ internal (degree-2) nodes and $L(T)$ leaves, then $N(T) + 1 = L(T)$. (See appendix B for the definition of "full binary tree").

3. Prove that $n \not\in o(10n)$ using the $c$ and $n_0$ style definitions.

4. Prove that $n \in o(n \log n)$ using the $c$ and $n_0$ style definitions.

5. Exercise 3.2-3 on page 60 (or page 57 of 2nd edition) on factorial asymptotics.

6. Find a pair of monotonically increasing functions $f(n)$ and $g(n)$ such that $\lim_{n \to \infty} f(n) = \infty$, $\lim_{n \to \infty} g(n) = \infty$, $f(n) \not\in O(g(n))$ and $g(n) \not\in O(f(n))$. (This one can be very tricky)

7. Problem 4.3.1 on page 96: Use master theorem method to get tight asymptotic bounds for the following recurrences:
   - $T(n) = 2T(n/4) + 1$
   - $T(n) = 2T(n/4) + \sqrt{n}$
   - $T(n) = 2T(n/4) + n$
   - $T(n) = 2T(n/4) + n^2$