CMPS 201 First Regular Homework

Sept. 28, 2004, Seven problems, 35 points. Due Tuesday, October 5

The reading for this week is Section 31.2, and finish Chapters 1-4 and Appendices A and B. If you want to read ahead, read Chapter 7 (heaps and heapsort), Chapter 8 (Quicksort), and as much of chapter 5 and Appendix C (probability) as you need to understand the average case analysis of Quicksort.

1. (5 pts) Prove formally that for any fixed (i.e. constant) \( b > 1 \), the function \( \log_b n \) is in \( \Theta(\log_2 n) \). Use the \( c \) and \( n_0 \) style definition from the book.

2. (5 pts) Find a pair of increasing functions \( f(n) \) and \( g(n) \) (both from the positive integers to the positive integers) such that \( f(g(n)) \) is not in \( O(g(f(n))) \).

3. (5 pts) Find a pair of monotonically increasing functions \( f(n) \) and \( g(n) \) such that \( \lim_{n \to \infty} f(n) = \infty \), \( \lim_{n \to \infty} g(n) = \infty \), \( f(n) \notin O(g(n)) \) and \( g(n) \notin O(f(n)) \).

4. (10 pts) Problem 4-6 on page 87 (VLSI chip testing).

5. (10 pts) Prove by induction that:

   Any undirected graph having the same number of edges as nodes contains a cycle.

   (Hint: you may use without proof the fact that any connected graph on \( n \) vertices with \( n \) edges has at least one vertex of degree \( \leq 2 \), and consider replacing a node of degree 2 with a single edge in the inductive step.)

Other recommended problems (not to be turned in):
Prove by induction that if \( T \) is a non-empty full binary tree, then \( N(T) + 1 = L(T) \);
exercise 3.2-3 on page 57 (factorial asymptotics);
problem 3-3 part a) on page 58 (ranking functions);
problem 4.3.1 on page 75 (master theorem);
prove that \( n \notin o(10n) \) using the \( c \) and \( n_0 \) style definitions; and prove that \( n \in o(n \log n) \) using the \( c \) and \( n_0 \) style definitions.