Beer Drinkers
List all drinkers who frequent every bar that serves AMSTEL
relational algebra
\[ \text{frequents} \div \pi_{\text{bar}}(\sigma_{\text{beer}=\text{'AMSTEL'}}(\text{serves})) \]
TRC
\[ \{t|(\forall u \in \text{serves})(u[\text{beer}] = \text{'AMSTEL'} \rightarrow (\exists w \in \text{frequents})(w[\text{drinker}] = t[\text{drinker}] \land w[\text{bar}] = u[\text{bar}])\} \]
DRC
\[ \{x|(\forall y)(\forall z)((y, z) \in \text{serves} \rightarrow (x, y) \in \text{frequents})\} \]
Semijoin
\[ \pi_{\text{R.A},\text{R.B},\text{R.C}}(\sigma_{\text{R.B}=\text{S.B} \land \text{R.C}=\text{S.C}}(\text{R} \times \text{S})) \]
Explain why \( R - S \) cannot be expressed using the other four basic relational algebra operations.
Quick Answer: The other four operations are monotone, while \( R - S \) is not monotone.
Definition: an operation \( F \) on relations with arguments \( R_1, R_2, \ldots, R_k \) is monotone if whenever \( R_1 \subseteq R'_1, R_2 \subseteq R'_2, \ldots, R_k \subseteq R'_k \), then \( F(R_1, R_2, \ldots R_k) \subseteq F(R'_1, R'_2, \ldots, R'_k) \).
Fact 1: Each of the operations \( \cup, \times, \sigma, \pi \) is monotone (follows easily from the definitions).
Fact 2: \( - \) is not monotone. For instance, \( R - \emptyset = R \), but \( R - R = \emptyset \) (i.e. we increased the second argument and got a smaller result).
Fact 3: Compositions of monotone operations are monotone (proved using induction).
Hence, every relational algebra expression built using \( \cup, \times, \sigma, \pi \) is monotone so it cannot express \( - \).