Problem set 2

1) For the bargaining game done in class on Thurs, write out the EFG and SFG of each. Solve by BI, IDSDS or IDWDS, if possible. State why or why not for each solution concept.

2) Question 3.7 out of Harrington.

3) Consider the following extensive form game

\[\begin{array}{c}
\text{1} \\
\text{2} \hspace{1cm} \\
\text{(1,2)} \hspace{1cm} (\text{-2,10}) \hspace{1cm} (\text{-3,5}) \hspace{1cm} (\text{2,-2})
\end{array}\]

a) What is the strategy space for players 1 and 2?
b) Write the strategic form of the game.
c) Can you arrive at a unique strategy profile with backwards induction? If so, what is it?
d) Using the normal form of the game, apply iterated dominance. Does the procedure find a unique strategy profile that “solves” the game? If so what is it?

Now considered the modified game:

\[\begin{array}{c}
\text{1} \\
\text{2} \hspace{1cm} \\
\text{(1,-2)} \hspace{1cm} (\text{-2,10}) \hspace{1cm} (\text{-3,5}) \hspace{1cm} (\text{2,-2})
\end{array}\]

e) Write the strategic form of the game.
f) Using the strategic form of the game, apply iterated dominance. Does the procedure find a unique strategy profile that “solves” the game? If so what is it?
4) Harrington Chapter 3 question 10.

5) In Cournot duopoly, two producers of an identical product (call them firms A and B) simultaneously choose how much of that product to produce, say \( q_A \) and \( q_B \), between 3 and 7. For simplicity, assume that production cost is 5 per unit. The price \( p \) is \( 20 - (q_A + q_B) \). This is because price falls as the supply increases. Payoffs are profit = (price – unitcost)*quantity, e.g., \((20 - q_A - q_B - 5) q_A\) for firm A. To simplify the game assume that the only available \( q \)'s are 3, 5, and 7.
   a. Write out the extensive form for this game.
   b. Now write out the extensive form for the Stackelberg variant of this game, in which firm A chooses first, and firm B observes \( q_A \) before making its own choice.
   c. Write out the normal form for the simultaneous move version of the game. For the Stackelberg version of the game, what is specification of B’s strategy? How big is the table describing this game’s normal form?
   d. For the original simultaneous choice version of the game, can you identify one or more Nash equilibria? If so, what is it (are they)?

6) Harrington Chapter 4, question 2
7) Harrington Chapter 4, question 5
8) Harrington Chapter 5, problem 5
9) You and your 9 closest friends are each trying to decide whether to buy a Zbox or a Vii video game console. You and 4 of your friends like the Zbox more than the Vii. The other 5 like the Vii better. However, the more of your friends that have the same console as you, the better off you are since you can share games with more friends. Each player carries an index \( i \in \{1,...,10\} \). Players 1 though 5 are the ones that are partial to Zbox while players 6 through 10 are the ones partial to the Vii. Let \( x_i \in \{Z, V\} \) be player \( i \)'s strategy. We also define the “indicator function” \( 1(x_i = V) = 1 \) if \( x_i = V \) and otherwise it is equal to 0. Similarly \( 1(x_i = x_j) = 1 \) if \( x_i = x_j \) otherwise it is equal to 0. Thus, the expression \( \sum_{j=1}^{10} 1(x_i = x_j) \) counts how many players have the same game console as player \( i \) (including player \( i \) himself).
   The payoffs are
   
   for \( i \in \{1,...,5\} : \ U_i = 10 \cdot 1(x_i = V) + 3 \cdot \sum_{j=1}^{10} 1(x_i = x_j) , \)
   
   for \( i \in \{6,...,10\} : \ U_i = 7 \cdot 1(x_i = Z) + 3 \cdot \sum_{j=1}^{10} 1(x_i = x_j) . \)
   
   Find any and all Nash equilibria of this game. Are any equilibria payoff dominant? Does either the term “tipping game” or “congestion game” describe this game well?