**Instructions:** Please try these problems by yourself, with a 105 minute time limit. Then discuss them with your teammates and other students. Finally, check your answers against those posted on the class website the day before the midterm exam. Do **not** turn in your answers; these are just for practice!

**Note:** These problems emphasize material not on any of the problem sets, e.g., that covered in Chapter 8 of Harrington. The actual midterm exam will include more material from earlier chapters and lectures.

1) You, player A, are engaged in a team class project with your buddy player B. You each have to decide what level of work to put into the project, and your choices are 1-low, 2-medium, 3-high. Your work is designated \( w_a \) and that of your buddy \( w_b \). The grade on the project is determined by the total work \( W = w_a + w_b \) as

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(You will notice that your grade has diminishing returns as the amount of work goes up.) Each player i’s payoff is \( \text{Grade} - 10w_i \).

a) Suppose you agree with your buddy to work independently. You will just staple your two parts together on the day it is due, without observing how hard your partner worked. Express the game’s strategic form as a bimatrix.

b) What Nash equilibria can you find?

c) What is the equivalent extensive form of the game you analyzed in parts a and b?

d) Now suppose you decide that you, player A, will finish your part and then pass it to B. B will see how hard you worked, and then decide how hard to work on their part. What is the extensive form of this game?

e) Can you find the subgame perfect Nash equilibrium to the game of part d? Based on this analysis would you want to be the player who works on their part first or last?

2) Player #1 (the Serf) first chooses either to plant crops (P) or to hide the seed (H). If he chooses P, then at harvest time Player #2 (the Duke) either takes the entire crop (T) resulting in payoffs \((0, 3)\) for the two players, or else shares (S) resulting in payoffs \((2, 2)\). If the Serf chooses H, then the game ends with payoffs \((1, 0)\).

a) Draw the tree (i.e., the extensive form) for this 2 move game.

b) Find the subgame perfect Nash equilibrium this game, and the corresponding payoffs.

c) Write the game in normal form, and find **all** Nash equilibria.

d) Find strategies (not necessarily best responses) that lead to an efficient outcome, i.e., one that maximizes the payoff sum.

e) Consider the following “pre-play” move for the Duke. He chooses in advance the tax rate \( t \), the fraction \( t \) of the crop he can take. (An
independent legal system* ensures that he can't change his mind later, but for simplicity, don’t need to include this as an additional player.) Draw the tree for this 2 player, 3 move game, and try to find a value of \( t \) for which the subgame perfect Nash equilibrium is efficient.

3) Congratulations! You have been appointed to be the Car Czar of the United States. You are responsible for recommending a reorganization of the industry to reduce the losses the carmakers are suffering. Your demand forecaster tells you that you should think of all cars as being identical, and that the market price \( p \) for cars depends on the total quantity \( Q \) produced in the following way:

\[
p = (30 - Q).
\]

Suppose there are \( n \) car companies, and each company chooses a quantity \( q_i \) of cars to produce. It costs each car company 10 to make each car, and each car company has fixed “legacy” costs of 25 so that the payoff function for each company is

\[
V_i = (p - 10)q_i - 25.
\]

(Note on units: The prices are in units of thousands of dollars, quantities in units of millions of cars, and payoffs in units of billions of dollars. The formulas above are consistent with this, so you can work with the formulas as given without adding lots of zeros to the end of all the numbers!)

a) What is the best response function of company \( i \)? Express your answer in terms of \( Q_i \), the sum of the quantities the companies other than \( i \) produce.

b) Find a symmetric NE.

c) What is the profit of each company in the equilibrium you find in part b?

d) The president wants as much competition as possible while having an industry that doesn’t lose money. What is the largest number of companies in the car business possible and still have each company not lose money (have a nonnegative payoff)?

4) Harrington Chapter 8, Question 7. If you have time, also try Questions 11 and 12.

*Many economic historians think that the emergence of such enforcement is a key reason for the rapid growth of Western Europe 1500-1900.