1. Captain Picard is about to beam down to the planet Romulus to “negotiate” a treaty. He has heard conflicting reports that the Romulans are a bit surly, but he is not sure whether they are simply Testy (e.g., Peevish) that day or Murderous that day. Aggression is the Romulan nature and even if they are Testy they fully expect Picard to defend himself (depending on the level of their own attack). Picard has to be prepared for every eventuality. He talks to Number One before heading down to look up some probability stats on Romulans. Number One reports that 8 out 10 diplomats report meeting a Testy Romulan after culminating the negotiations. However, 2 out 10 diplomats never returned from the planet after negotiating (we can safely assume that they are dead and the Romulans were Murderous). This is a most important treaty for the United Federation, but Picard also values his life. He has to decide whether to set his phaser to Stun or to Kill to handle a Testy Romulan or Murderous one. The Romulans will either choose to accept the negotiated settlement (No War) or go to war (War) with the United Federation of Star Systems (based on Picard’s actions). (Picard will of course withdraw from the planet after either stunning or killing the Romulans). If Picard is too weak with a Murderous Romulan the payoffs differ than if he is too strong with a Testy Romulan. Picard will draw his weapon and fire no matter what. The problem is that Picard is a bit senile in his advancing years, and Number One reminds him that even if he sets his phaser to Stun or Kill before he leaves, he has developed a well-known (well known even to the Romulans) nervous tick in which while fiddling with his phaser he randomly sets it to stun with probability .5 or to kill with probability 0.5. Picard simply can’t help himself. Picard’s payoffs are clear and the Romulans know them:

- If Picard has his Phaser set to Stun, Picard prefers a negotiated settlement with the Romulans (No War, payoff of 4) to War (payoff 1, in which case his mission has been a total failure).
- If Picard has his Phaser set to Kill, Picard prefers a negotiated settlement from the Romulans (No War, payoff of 3) to War (payoff 2). [N.B., We all know that Picard prefers a less aggressive response (phaser set to Stun with payoff of 4) over a more aggressive response (phaser set to Stun with payoff of 3, above). The United Federation would prefer that Picard be as pacifistic as possible with the Romulans to reinforce any future treaties.

Picard’s payoffs are invariant to (do not change with) the mood of the Romulans. Nevertheless, Picard knows the payoffs for a Testy Romulan or a Murderous Romulan (based on Number One’s information above):

- The payoffs for the case of a Testy Romulan are clear (to both the Romulans and Picard). They prefer No War (4) to War (3) in the case of Picard using his phaser set to stun on the Romulan diplomat. Naturally the Romulans prefer to Wage War (2) to signing the negotiations (1) in the case of Picard killing the Romulan diplomat with his weapon set to Kill.
- The payoffs for a Murderous Romulan are likewise clear. They prefer War (4) to No War (2) if Picard uses his phaser on stun. They prefer War (3) to No War (1) in the case of Picard killing their negotiator with his weapon set to Kill.

a) Faced with the Romulan negotiations, how should Number One proceed when he prepares an EFG for Picard to use in deciding upon a strategy? Draw the EFG (5 pts).
This EFG is very similar to the one for Kirk/Gorn or Chamberlain Hitler with a simple twist whereas Kirk or Chamberlain faced a simple decision node, this is now a Nature move for Picard with probability $p=0.5$ for Stun and $p=0.5$ for Kill. Therefore this is now information on the rationale behavior of Picard that Romulans lack, and both players have information sets (things they do not know). I also changed the preferences of Kirk, a man of action and passion, to those of Picard, a man of contemplation and avoidance of aggression, in keeping with their characters, so the payoffs will differ.

![Game Tree Diagram]

I did not ask for the solution of the EFG, but it would be very similar to the gunslinger example from lecture, if not a bit tedious.

b) Just before Picard gets in the transporter, Number One realizes that he can either fuse the Phaser to Stun or fuse it on the Kill setting to eliminate Picard’s well-known nervous tick. Draw the new EFG (5 pts), then the SFG (5 pts).

We assume of course that all facts are known to both Romulans and the Federation (Number One has hailed the Romulans and let them know what he has done). Number One is rational and acts in the interests of Picard (who is really only acting in the interests of the Federation, and its payoffs).

This game tree is now identical to Kirk/Gorn (see HW solution) with minor changes to the payoffs of Picard (from Kirk), but Romulan payoffs stay the same as Gorn.

(just drop the Nature move on Romulans)

The SFG is thus similar with a matrix for murderous ($p=0.2$), and testy ($p=0.8$), but the form of the matrix is the same as the HW solution.
c) Solve for Picard’s expected payoffs (5 pts) and figure out which setting Number One fused on Picard’s phaser and force Picard’s optimal strategy (5 pts).

Notice that for Picard Stun SD Kill (this differs slight from Kirk Gorn)
For the Romulans however, they greatly prefer W if murderous, that is W SD n-W for them.

Picard’s payoffs for Stun are computed similarly to Kirk’s in the HW:
\[0.8 \times 3 + 0.2 \times 1 = 2.6\]
Picard’s payoffs for Kill are computed similarly to Kirk’s in the HW:
\[0.8 \times 2 + 0.2 \times 2 = 2.0\]

Number one should force Picard’s phaser to Stun and inform the Romulans that he has set the phaser (but not what setting he has used). The romulans can of course compute what setting he will use, by the math above and the full disclosure of the game tree in b.

d) Why was it important that Number One intervene and set the phaser setting for Picard? (5 pts). Simply describe the difficulty for the Romulans.

Various answers are acceptable here provided they have a game theoretic spin. For example the Romulans, not knowing that Picard (a senile Picard mind you) will not act rationally means their payoffs are more difficult to compute (along the lines of gunslinger), but still doable for them.

Just an aside, Number One realizes he has to intervene because Picard is not going to maximize the payoffs to the Federation, it will be a waited average payoff, not the simple payoff maximization (as observed in the Gorn/Kirk encounter).

2. In 1804, sitting Vice President Aaron Burr challenged Alexander Hamilton, former Treasury Secretary, to a gun duel. (H)amilton needs to decide whether to (A)ccept or (R)eject the challenge. Hamilton notifies Burr of his decision by letter. Rejecting the challenge will show Hamilton to be a coward, giving him a payoff of 0 and (B)urr a payoff of 10. At that point the game would end.

If Hamilton (A)accepts the challenge, the two will meet in Weehawken, New Jersey just across the Hudson river from New York City. The two men will simultaneously decide whether to shoot to (K)ill or shoot the (G)round. If both shoot the ground, they each get a payoff of 5 because both have the honor of having participated in the duel without getting killed. If Hamilton shoots the Ground and Burr shoots to Kill, then Hamilton dies, giving Hamilton a payoff of -1 and Burr a payoff of 2. (Burr’s payoff isn’t higher because committing murder is bad for one’s political career, even in 19th century America!) In the reverse situation, when Burr shoots the Ground and Hamilton shoots to Kill, the payoffs are 2 for Hamilton and -1 for Burr. Finally, if both men shoot to Kill, there’s some probability that one or both men die, so we take the expected payoff to be 1 each in this scenario. [40 points]

a) Draw the Extensive form game tree [5 points]
b) Consider the subgame after (H)amilton accepts. Write out the bimatrix in Normal form. Note: Make H the row player. [5 points]

<table>
<thead>
<tr>
<th></th>
<th>G</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>(5,5)</td>
<td>(-1,2)</td>
</tr>
<tr>
<td>K</td>
<td>(2,-1)</td>
<td>(1,1)</td>
</tr>
</tbody>
</table>

Burr

\[ \begin{bmatrix} 
G & K \\
H & (5,5) & (-1,2) \\
K & (2,-1) & (1,1) 
\end{bmatrix} \]

c) Find all Nash equilibria of the subgame you considered in part B. For any equilibria you find, identify the expected payoff for each player. Note: If you need to consider mixed equilibria, Use p for the probability H plays G and q for the probability B plays G. [10 points]

(G,G) and (K,K,) are both pure strategy Nash Equilibria. There is also a mixed equilibrium. Let q be the probability Burr plays G. For Hamilton to be indifferent between actions it must be that

\[ 5q + (-1)(1-q) = 2q + (1-q) \]
\[ 6q - 1 = q + 1 \rightarrow q = 2/5 \]

Similar analysis yields that p=2/5

Thus the mixed equilibrium is where H and B each pick G w.p. 2/5 and K w.p. 3/5

d) What is the expected payoff of each player in each equilibrium of the subgame you considered in parts B and C [5 points]?

The (G,G) and (KK) eqs. give payoffs of (5,5) and (1,1) respectively. The mixed eq. gives a payoff of 5q + (-1)(1-q) = 2q + (1-q) = 7/5 to each player.

e) Identify all subgame perfect Nash equilibria (SPE) that you find. Be sure to specify the complete strategy for each player. [10 pts]

If the players expect the (1,1) payoff, eq in the subgame, then Hamilton will pick R in his first decision node. Thus this SPE is

Hamilton: R/K  Burr: K

If the players expect the (5,5) payoff, eq in the subgame, then Hamilton will pick A in his first decision node. Thus this SPE is

Hamilton: A/G  Burr: G
If the players expect the (7/5, 7/5) eq. in the subgame, Hamilton will also pick A in his first decision node. Thus, this SPE is

Hamilton: \((G\ w.p.\ 2/5,\ K\ w.p.\ 3/5)\) \quad Burr: \((G\ w.p.\ 2/5,\ K\ w.p.\ 3/5)\)

f) Suppose that Hamilton were to get a payoff of 1.5 instead of 0 from rejecting the duel, and all other payoffs are the same as above. Identify all subgame perfect Nash equilibria (SPE). [5 pts]

There are still 3 SPE:

If the players expect the (1, 1) payoff, eq in the subgame, then Hamilton will pick \(R\) in his first decision node. Thus this SPE is

Hamilton: \(R/K\) \quad Burr: \(K\)

If the players expect the (5, 5) payoff, eq in the subgame, then Hamilton will pick \(A\) in his first decision node. Thus this SPE is

Hamilton: \(A/G\) \quad Burr: \(G\)

If the players expect the (7/5, 7/5) eq. in the subgame, Hamilton will pick \(R\) in his first decision node. Thus, this SPE is

Hamilton: \(R/(G\ w.p.\ 2/5,\ K\ w.p.\ 3/5)\) \quad Burr: \((G\ w.p.\ 2/5,\ K\ w.p.\ 3/5)\)

3. The objective of Goldmine National Park (GNP) is to make as much money from tourists as possible. GNP offers (L)uxury hotel rooms and (T)ent cabins.

It is known that potential customers are either (S)tudents, (M)iddle class, or (R)ich income with equal probability. A marketing study found that the utility each type gets for staying in each type of accommodation:

<table>
<thead>
<tr>
<th></th>
<th>Tent Cabin</th>
<th>Luxury</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students</td>
<td>$76</td>
<td>$110</td>
</tr>
<tr>
<td>Middle Class</td>
<td>$101</td>
<td>$200</td>
</tr>
<tr>
<td>Rich</td>
<td>$149</td>
<td>$702</td>
</tr>
</tbody>
</table>

The net payoff is the customer’s utility minus what they pay.

Potential customers who choose not to stay in the park at all get a payoff of $0.

A marketing consultant tells GNP that they should choose one of the following 3 pricing strategies

i) Luxury: $700, Tent $100

ii) Luxury: $650, Tent $100

iii) Luxury: $625, Tent $75

We neglect the cost to provide each type of accommodation, and just assume that GNP’s payoff is revenue.

Since potential customers can hide their income levels from GNP when they reserve accommodation, when a customer arrives at GNP’s web site, the GNP does not know what type of customer (s)he is.

Not knowing the type of customer, GNP either presents pricing plan (i), (ii) or (iii). The customer then selects from (N)ot stay, (T)ent cabin, (L)uxury. [30 points]
a) Consider the game tree below:

```
Is this game tree correct? If yes, say so. If not, please correct it. [5 points]
It is incorrect. All of the GNP nodes should be grouped in an information set.
```

b) What is the strategy space for GNP [5 points]?

```
{price plan i, plan ii, plan iii}
```

c) Find a perfect Bayesian equilibrium [10 points]

We proceed by figuring out the payoffs of each player type if presented with a given price plan.

**STUDENTS:**

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T</td>
<td>-24</td>
<td>-24</td>
<td>1</td>
</tr>
<tr>
<td>L</td>
<td>-690</td>
<td>-640</td>
<td>-615</td>
</tr>
</tbody>
</table>

**MIDDLE CLASS:**

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T</td>
<td>1</td>
<td>1</td>
<td>26</td>
</tr>
<tr>
<td>L</td>
<td>-500</td>
<td>-450</td>
<td>-425</td>
</tr>
</tbody>
</table>

**RICH:**

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T</td>
<td>49</td>
<td>49</td>
<td>74</td>
</tr>
<tr>
<td>L</td>
<td>2</td>
<td>52</td>
<td>77</td>
</tr>
</tbody>
</table>

So plan I gets Middle Class and Rich to take the tent and no one takes luxury. Thus the expected payoff for this strategy is $100 \times (2/3) = 66.666$

Plan 2 gets MC to take tents and Rich to take luxury. Expected payoff is $100 \times (1/3) + 650/3 = 250$
Plan 3 gets students and MC to take tents and rich to take luxury. Expected payoff is $75*(2/3) + 625*(1/3) = 775/3$.

Thus plan 3 gives the best expected payoff to GNP, so it must be the one it picks. Thus the PBE is:

- **GNP:** Plan iii
- **Students:** $N$ if plans i or ii, $T$ if plan iii
- **Middle Class:** $T$ for all 3 plans
- **Rich:** $T$ if plan 1, $L$ if plans 2 or 3

(Full strategies must be specified!)

d) For the equilibrium you found in part (a) compute the expected payoff of GNP in its interaction with a potential customer. [5 points]

> From above, it’s $775/3$

e) Comment whether the potential customer’s strategy in your equilibrium is pooling, separating, or semi-separating. [5 points]

> It is semi-separating

4. Consider the following game [30 points]

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>Y</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8,8</td>
<td>3,7</td>
<td>5,4</td>
</tr>
<tr>
<td>B</td>
<td>1,3</td>
<td>4,2</td>
<td>6,2</td>
</tr>
<tr>
<td>C</td>
<td>1,4</td>
<td>2,3</td>
<td>4,2</td>
</tr>
<tr>
<td>D</td>
<td>7,3</td>
<td>5,5</td>
<td>1,2</td>
</tr>
</tbody>
</table>

a) Find all NE (pure and mixed). [10 points]

b) In the solution of the NE, you generated a reduced game, is this game symmetric? [5 points]

c) Which of these NE are also ESS (if any) and why (state the conditions for the ESS and how it differs from NE). [10 points]

d) Can you describe the evolutionary dynamics of this game if this were a 2-player game played in a single population (a graph is essential here, along with the equation describing the difference in fitness functions plotted on the graph). In other words, solve for the equilibrium point, and describe what happens to $p$, the frequency of one strategy above versus below the equilibrium point. [5 points]

a) Solution

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8,8</td>
<td>3,7</td>
<td>5,4</td>
</tr>
<tr>
<td>B</td>
<td>1,3</td>
<td>4,2</td>
<td>6,2</td>
</tr>
<tr>
<td>C</td>
<td>1,4</td>
<td>2,3</td>
<td>4,2</td>
</tr>
<tr>
<td>D</td>
<td>7,3</td>
<td>5,5</td>
<td>1,2</td>
</tr>
</tbody>
</table>
in the above matrix, for player 1: strategy C is SD by A, leaving:

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8,8</td>
<td>3,7</td>
<td>5,4</td>
</tr>
<tr>
<td>B</td>
<td>1,3</td>
<td>4,2</td>
<td>6,2</td>
</tr>
<tr>
<td>D</td>
<td>7,3</td>
<td>5,5</td>
<td>4,2</td>
</tr>
</tbody>
</table>

in the above matrix, for player 2: strategy Z is SD by X, leaving:

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8,8</td>
<td>3,7</td>
</tr>
<tr>
<td>B</td>
<td>4,3</td>
<td>4,2</td>
</tr>
<tr>
<td>D</td>
<td>7,3</td>
<td>5,5</td>
</tr>
</tbody>
</table>

b) in the above matrix, for player 1, Strategy B is SD by strategy D, leaving a symmetric 2x2 game:

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8,8</td>
<td>3,7</td>
</tr>
<tr>
<td>D</td>
<td>7,3</td>
<td>5,5</td>
</tr>
</tbody>
</table>

This matrix has two NE (bold). Solved by mutually rational BR (above).

c) Given that this game has two pure NE it cannot have an ESS. Why? Write out the 2x2 symmetric game and note that:

strategy A in player 1 = strategy X in player 2, and strategy D in player 1 = strategy Y in player 2 to yield a simple matrix from the bimatrix game:

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

(Note: you can solve the bimatrix for p and q but this is much harder and overkill since it is a symmetric game, you need only solve the strategies for player 1 and by symmetry this will apply to player 2).

To be an ESS Thomas (1985) states that
condition 1) $E(S,S) \geq E(T,S)$ – the condition for a NE for strategy S, it is the best response AND
condition 2) $E(S,T) > E(T,T)$ – S can invade when S is rare and T is common.

Each NE satisfies the first condition of Thomas’ (1985) definition of the ESS: Namely:

$E(A,A) > E(D,A)$ (i.e., 8 > 7),
which satisfies condition 1, A has high fitness when common, so A is potentially an ESS, but that:

$E(A,D) > E(D,D)$ (i.e., 3 > 5 is not true), A cannot invade when rare. which violates condition two.

The converse of course is true for $E(D,D) > E(A,D)$, it satisfies condition 1 but that $E(A,A) > E(D,A)$ violates condition 2 for $E(D,D) > E(A,D)$

BELOW I consider the mixed NE. Full marks only require an analysis of the ESS conditions for the pure equilibria.
d) Given this is a symmetric game it suffices to see if there exists a mixed NE for one player. We can rewrite this matrix as a simple matrix and solve for the equilibrium:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

So \( F(A) = 8p + 3(1-p) \)
and \( F(D) = 7p + 5(1-p) \)

And \( F(A) - F(D) = d(p) = p - 2(1-p) = p - 2 + 2p = 3p - 2 \)

Setting this equal to zero gives us the crossing point:
\( p = \frac{2}{3} \) is the equilibrium point where strategies A and D have equal fitness.

If you draw the graph of the function \( d(p) \) you will see it gives low fitness to strategy A when \( p < \frac{2}{3} \) so that A will decrease (and D will increase) and it gives high fitness to A when \( p > \frac{2}{3} \) so A will increase. In other words the equilibrium \( p = \frac{2}{3} \) is not stable and you should draw arrows of flow away from \( \frac{2}{3} \) down to zero and above \( \frac{2}{3} \) up to 1.

I do not expect the students to carry out the ESS analysis for the Mixed NE, but for completeness, here is a brief solution.

To carry out the ESS analysis of the mixed NE, the \( F(A, D) \) functions are required. It can easily be shown that to either side of the equilibrium \( p \), any small perturbation \( p + \Delta p \) will take the population away from the mixed NE to either side of the equilibrium. This behavior violates the stability condition 1 of Thomas definition. E.g., when common the NE is a best response (as good as or equal to any other strategy).

This can be shown with the \( d(p) \) function (which considers each strategy when common to either side of the equilibrium).
\( d(p) = 3p - 2 + \Delta p \) reflects the inequality \( E(A, A) > E(D, A) \) as \( 0 > E(D, A) - E(A, A) \) (e.g., negative quantities satisfy condition 1). Notice from the graph you draw that the \( d(p) = 3p - 2 + \Delta p \) violates condition 1, when \( \Delta p \) is either \( >0 \) or \( <0 \). (this is a property of a Stag hunt game which the 2 strategies \( \{A, D\} \) satisfy. This is the other 2x2 case not considered in the HW, but involving the same soln).

5) Imagine that there are only 2 producers of widgets in the world, A and B. Firm A and B have to decide how many widgets \( q_{ak} \) and \( q_{bk} \) to produce in each year \( k \). The price of widgets in each year is \( (9 - q_{ak} - q_{bk}) \). Widgets cost nothing to produce, so firm A and B get payoffs of \( p_{ak} = (9 - q_{ak} - q_{bk}) q_{ak} \) and \( p_{bk} = (9 - q_{ak} - q_{bk}) q_{bk} \) respectively.

A) Suppose the firms were going to play this game for one year and then the game ends. (Both firms know the game is one shot). What is the Nash equilibrium quantity of widgets each produces. In NE what is each firm’s payoff? (You will find a symmetric equilibrium.)

\[
dp_{at}/dq_{at} = 9 - 2q_{at} - q_{bt} \rightarrow q^*_{at} = (9 - q^*_{bt})/2
\]

Similarly, \( q^*_{bt} = (9 - q^*_{at})/2 \)
\( q^*_{at} = (4.5 + 0.5 q^*_{at})/2 \) \( \rightarrow q^*_{at} = 3, q^*_{bt} = 3 \)

payoff = \( (9 - 6)3 = 9 \) for each firm
B) Suppose now that the firms were going to play the game for exactly 2 years and both firms know this. What is the Nash equilibrium quantity of widgets each produces in each year?

Each firm produces 3 in the last year since it’s one shot. By backwards induction each firm produces 3 in year 1 as well.

C) Suppose the two firms could collude and maximize the sum of their payoffs in any given year $k$:

$$p_{ak} + p_{bk} = (9 - q_k)q_k \quad \text{where } q_k = q_{ak} + q_{bk}.$$  

In particular, suppose they choose $q_k$ to maximize $p_{ak} + p_{bk}$ then they each agree to produce $q_k/2$. How much would each firm produce in this scenario? What are their payoffs in this scenario?

$$\frac{dp_k}{dq_{kl}} = 9 - 2q_k \rightarrow q^*_{k} = \frac{9}{2}. \quad \Rightarrow \text{each firm produces } \frac{9}{4}$$

payoffs = $(9 - \frac{9}{2})\frac{9}{4} = (9/2)(9/4) = 81/8 = 10 \frac{1}{8}$ for each firm

D) Suppose that each firm now must pick from 2 quantities, the amount they’d produce in part A, which we now call “High” and the amount they’d produce in part C, which we now call “Low.” [Hint: if your answer for C isn’t lower than your answer for A, go back and check your work!] Also suppose that we are back in the “one-shot” setting in which the firms are only concerned with the first year. Write out the payoff matrix of the NFG.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>$(10.125, 10.125)$</td>
<td>$(8.4375, 11.25)$</td>
</tr>
<tr>
<td>H</td>
<td>$(11.25, 8.4375)$</td>
<td>$9, 9$</td>
</tr>
</tbody>
</table>

E) Suppose the firms now repeat the above game forever. For what values of the discount factor $d$, would it be a subgame perfect Nash equilibrium for each firm to play “grim trigger” strategies that result in each firm always producing L.

The net payoff from defecting in period $k$ is $9/8$ above sticking to L. After that, they loose $9/8$ forever (compared to sticking to L). The NPV is 0 when

$$\frac{9}{8} - \delta(1 + \delta + d^2 + \ldots)(9/8) = 0$$

$$9/8 - \delta/(1 - \delta)(9/8) = 0$$

$$1 - \delta/(1 - \delta) \Rightarrow \delta = 1/2$$

Any $\delta \geq 1/2$ works.