Problem Set #2

Due in class Tuesday, October 11.
You are encouraged to discuss all problems with other members of your group, and other class members. Please turn in your own individual writeup of problems in Parts I and II. For Part III, please turn in only one copy for the entire group, with all members’ names written down.

Part I. Word Problems.

1. Two producers of an identical product (call them firms A and B) choose how much of that product to produce, say \( q_A \) and \( q_B \). Assume that production cost is 6 per unit and that price \( p \) is \( 30 - q_A - q_B \). Payoffs are profit = (price – unitcost)\(^\times\)quantity, e.g., \((30 - q_A - q_B - 6)q_A\) for firm A. To keep things a bit simpler, assume that the only allowable choices for \( q_A \) and \( q_B \) are 4, 8, 12 and 16.
   a. In Cournot duopoly, both firms chose simultaneously. Write out the extensive form for this game.
   b. Now write out the extensive form for the Stackelberg variant of this game, in which firm A chooses first, and firm B observes \( q_A \) before making its own choice.
   c. Write out the normal forms for both games.
   d. Eliminate strictly dominated strategies in both games. Iterate. Did you find a unique solution?

Extra credit problem.
The professor of a MWF class announces that she will give quiz some day next week, but the particular day (M, W, or F) will be a surprise. A student argues that surprise is impossible: if the quiz is on Friday, it will not be a surprise since no other options remain. So it can’t be Friday. But in that case, it can’t be on Wednesday because that wouldn’t be a surprise given that it can’t be Friday. But now Monday won’t be a surprise either, since Friday and Wednesday have been ruled out. The student concludes that there will be no exam and doesn’t study. [Here’s what actually happened. The professor gave the quiz on Wednesday and the student was unpleasantly surprised!]

Philosophers and logicians have puzzled over this apparent paradox. Resolve the paradox by writing out a two player, zero sum game in which player #1, the Professor, chooses the day in advance, and player #2, the Student, guesses each day before class whether or not the exam is today (T) or later (L). Say the payoff is +1 to the Student and -1 to the Professor each time the student guesses correctly, and the opposite each time the Student guesses incorrectly. (For even more extra credit, see if you can actually solve this game.)

Part II. Problems from Harrington.
Write out your solutions to the following chapter-end exercises.
   Chapter 3: #3, 5, 7, 8.

Part III. Teams and Class Games.

Please turn over the page…
1. What is the name of your team? Who are the members, and what are their majors?
2. What term project ideas are currently under consideration by your team?
3. Please attach your record sheet for the Quadruped Game played in class Thursday, September 29. You will earn bonus points according to your total payoff on the sheets.
4. Analyze the Quadruped Game data, posted on the class website.
   a. Write down the full list of strategies for each player (I and II).
   b. In version A of the game, what fraction of the time overall was each strategy played? (The sum should be 1.0 or 100%).
   c. In version B of the game, what fraction of the time was each strategy played?
   d. Do you see any trends in these fractions or shares, when you compare the first few periods to the last few?
   e. If you had another period of each version game, what strategy would you choose? Why?